A MICROSIMULATION MODEL TO ESTIMATE ERRORS IN CROSS-SECTIONAL ESTIMATES OF LIFE EXPECTANCY IN DISABILITY

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1 Introduction

The assessment of expected duration of life of elderly individuals as healthy, with illness and/or with disability has become a task of increasing relevance for a number of reasons. First, accurate estimation of durations in various states is a strategic endeavor in the verification of theories regarding the predicted course of morbidity, disability and mortality. Second, evaluation of competing policies for elderly care is crucially dependent on accurate estimation of the demand size for and costs of health care and services for the dependent population. The latter cannot be attained without proper assessment of the expected durations and distribution of life of individuals by health statuses. With some notable exceptions, estimation of duration of time spent in various health statuses is carried out with cross-sectional information and with procedures resting on assumptions that may appear, on first blush at least, restrictive and confining. The one technique that most investigators, constrained as they are by the availability of cross-sectional information only, use with some regularity and success is the so-called Sullivan method. This procedure requires information about general mortality in the population and about the prevalence of a finite number of disabilities used to define, by themselves or in combinations, states of dependence or, more generally, health statuses that individuals may occupy. A less known, but quite reasonable approach, can be implemented at the expense of altering some assumptions and requiring additional information (Monteverde, 2004). We will refer to this as the Pseudo Multistate Model (PMM). As is the case for Sullivan' s method, this procedure requires information on overall mortality but, in addition, allows the investigator to specify parameters controlling the degree to which disabled and non disabled individuals are exposed to mortality differentials.Unlike the Sullivan procedures, PMM has the advantage of enabling the investigator to estimate indirectly all transitions rate of the underlying multistate process. These may be useful to estimate expected individual long term care costs and other useful actuarial calculations.

Although somewhat different in nature, both methodologies produce estimates of expected duration of life in health and disability under assumptions that appear to be quite strong. The first is the assumption of **stationarity**, namely, that transition rates governing the mortality and disability processes are constant over the recent past and, consequently, that the observed cross sectional rates are identical to the cohort rates. The second one is **homogeneity of risks**, an assumption that forces mortality risks of the disabled to be the same as that of the healthy. As mentioned before, this assumption can be relaxed somewhat in the second method introduced above. The third assumption is the **absence of recovery** (rate of recovery are zero), that is, those who become disabled cannot return to the healthy state. Users and critics of these procedures alike admit that, more often than not, one or several of these assumptions are not mer in most empirical applications (Rogers, Rogers and Belanger, 1989, 1990). Yet, despite some important work to identify the behavior of estimates (Robine and Ritchie, 1991; Robine and Mathers, 1993; Crimmins, Saito and Hayward, 1993;Lievre, Brouard and Heathcote 2003) there is no clear under-

standing about the direction, magnitude and distributional properties of the errors associated with departures from the three assumptions identified before. If we could confirm that these errors are small, the matter could be dismissed. In such case it would no longer be necessary to insist on requiring longitudinal information to estimate expectancy in health and disability when transition rates are changing over time or to collect additional (and costly) information when there is no basis to assert the absence of recovery or the lack of homogeneity of risks. Some authors have argued (Crimmins, Saito and Hayward, 1993) that Sullivan-like estimates reflect an assortment of conditions and that they are useful even under gross violation of assumptions but that, in any case, should be interpreted carefully rather than discarded altogether when some assumptions are known to be in error. However, with imperfect knowledge of sensitivity to errors it is difficult to understand the conditions under which Sullivan-type estimates adequately describe population health status conditions or when alternative measures are called for.

Our goal in this paper is to illustrate the magnitude, direction and distributional properties of errors generated as assumptions are violated. We show that, at least under the scenarios we propose, the magnitude of errors is modest but that, in some cases at least, should call for more than caution in interpretation. Although we focus only on violations to the assumptions of **homogeneity of risks and no recovery**, we suspect that when they are combined with lack of stationarity, errors could only grow and, consequently, make inferences more questionable. We also show that difficulties in obtaining unbiased estimates may become more serious if the investigator wants to do more than evaluating conditions at one point in time. The study of group differentials or, alternatively, the examination of time trends, for example, are equally prone to violation of assumptions and could also be affected by lack of robustness. The results we present below suggest that the average magnitude of errors is in general not large but that, under some scenarios at least, may exceed tolerable levels. If so, theory verification and estimates of costs of health care and services or actuarial calculations for decisions about insurance premia or insurance schemes feasibility could become problematic.

We use Monte Carlo simulation methods to generate individual health status trajectories that depend on well-defined and well-behaved transition rates. These rates are defined to generate scenarios departing to variable degrees from the assumptions invoked by Sullivan and the PMM methods. We then study the magnitude, direction and distributional properties of errors associated with each method. We focus on the differences between estimates of life expectancy in disability derived from the above mentioned methods and the "true" values embedded in each simulation. We then study the behavior of the absolute values of relative errors and their distribution.

2 Background

The most important rational for performing the exercise we propose in this paper is that the assessment of duration of life in disability or illness and in health is both theoretically and practically relevant. Indeed, verification of theoretical conjectures and evaluation of demand size and costs of long term care, for example, depend critically on the proper measurement of longevity and duration in various health states

2.1 Verifying competing theories

The nature of longevity and health dynamics continues to occupy a central piece in theories of aging. Two extreme and contradictory theories have been developed. The optimistic theory (the compression of morbidity) refers to a scenario where the onset of chronic illnesses is delayed and begins to affect mostly the last few years of life. The result is that healthy life is prolonged at a rate greater than that of total years of life. Consequently, the proportion of healthy life should be expected to increase (Fries, 1980). A more pessimistic scenario suggests that increases in average duration of life cannot be matched by corresponding improvements in the incidence rates and duration of morbidity (or rates of recovery). The outcome will be just opposite to the one suggested by the first scenario, namely, one where chronically ill persons are simply kept alive longer with the resulting expansion of morbidity (Gruenberg, 1977). Manton (1982) attempted to reconcile the optimistic and pessimistic theories by arguing that although increases in longevity may lengthen the duration of morbidity, they are also likely to decrease its severity and, consequently, the resulting "quality of life" of populations should improve.

Discriminating between these contrasting theoretical perspectives requires precise information on expected duration of life at later ages, duration of illness, disability by seriousness and information on recovery rates. Moreover, the procedures must be sufficiently robust to enable the investigator to assess changes over time as well as differential across social groups. The latter is strategic step as it permits to compare the results of conditions leading to potentially different courses of longevity, onset of illness, and disability and recovery. The controversy about declining disability in the US, for example, is not centered around large discrepancies in observed quantities but, quite the contrary, on somewhat fragile contrasts which could be due to real changes or to artifacts produced by period effects, slightly different definitions of disability, treatment of institutionalized populations (Freedman et al., 2004) or, alternatively, to violations of assumptions on which the basic measures are based.

2.2 Assessing size of demand and associated costs

Knowing currently or projected durations in different health status is the basis for estimating aggregate long term care and health services demand at the

population levels (Monteverde,2004; Mayhew, 2000). This information can be directly retrieved from Sullivan type of estimates. In addition, one may want more precise knowledge at the individual level about the exact timing of transitions for accurate estimates of present value of expected costs and insurance premia for individuals (Haberman and Pitacco, 1999). This information, however, *cannot* be retrieved from Sullivan type of estimates but may be caclulated from the output of procedures such as PMM. This justifies the implementation of alternative methods to those suggested by Sullivan, and this is the reason why in this paper we do not limit ourselves to examine behavior of Sullivan's procedure only. Whether the estimates are at the aggregate or individual levels, they are useful to forecasting the resources needed to treat ill individuals and to provide services to disabled people throughout the duration of their disability. The evaluation of aggregate or individual costs requires either estimates of expected duration in each health state or of transition rates. When these are combined with different types of services of long-tern care they yield a gross estimate of the resources required by one individual throughout his/her expected duration of life. Without the estimation of duration (or transition rates), this exercise is simply not feasible. It is quite obvious that the robustness of cost estimates is directly related to the robustness of estimates of expected durations in various health statuses (or transitions rates).Whether these calculations are needed for state based programs or to assess the feasibility of private insurance schemes, there is little alternative to the estimation of accurate measures of mean duration of life in various health status.

2.2.1 Methods of estimation

Alternative methods and models have been used to measure simultaneously mortality, disability and morbidity with the explicit objective to contrast empirically the conjectures identified above. Broadly speaking there are three different ways of doing so. First, there is the so-called Sullivan or prevalence-rate model (Sullivan, 1971). It is the simplest of procedures and the one requiring the least amount of information. The price one pays for its simplicity is to be at the mercy of the accuracy of some simplifying assumptions (see below). The second procedure is one that relies on cross sectional information to derive transition rates thus removing at least one of the assumptions on which Sullinvan's methods rests but introducing, in turn, a new assumption which may or may not be accurate (Rickayzen and Walsh, 2002; Monteverde, 2004). While these two methods are indirect ways to obtain estimates of expectancies, the third one is direct. This consists of the application of increment-decrement life tables to a simplified multistate model from which estimates of expected durations in various states can be readily obtained. In theory, this is the proper way of evaluating the parameters we are interested in but, as is well-known, estimation of increment decrement life tables requires knowledge of transitions rates at least at one point in time (Katz, Branch, Branson, Papsidero, Beck and Greer, 1983; Rogers, Rogers and Brach, 1989; Pollard, Golini and Milella,

1990; Rogers, Rogers and Belanger, 1989; Rogers, Rogers and Belanger, 1990; Land, Guralnick and Glazer, 1994; Robine and Mathers,1993; Lievre, Brouard and Heathcote 2003). Reliable estimation of multistate life tables needs large samples and longitudinal observation, requiring expensive study designs and have been implemented in only a few places. If nothing else but two waves of a panel study are available, the robustness of estimates associated with increment-decrement life tables will depend on the accuracy of the stationarity assumption (in addition to the size sampling variability). Table 1 identifies each method and the assumptions on which they are based.

Table 1 about here

Since restrictions on data collection are likely to persist for a long time in most countries in the world, it is important to have an idea, if only approximate, of errors associated with violations of assumptions underlying indirect methods of estimation of life expectancy in various health status. Knowledge of the magnitude, direction and distributional properties of errors and the conditions under which they are produced can help mitigate interpretational problems or, at the very least, lead to inferences that are explicitly contingent on known or estimated levels of uncertainty. This will enable us to assign adjusted weight to evidence invoked to confirm or reject interpretations about the course of morbidity and mortality, on the one hand, and to design more realistic cost scenarios to project long term care or health services needs, on the other.

2.3 Previous research on evaluation of methodologies

Ours is by no means the first nor the most original attempt to examine the matter of errors associated with simplified procedures to assess expected duration of life in various health status.However, despite a large number of studies

investigating the properties of various estimators, there is no comprehensive evaluation of comparative robustness of alternative methods to violations of all the three basic assumptions identified above. For example, while some studies explictly address problems posed by lack of stationarity, they consider neither the relevance of mortality differentials (homogeneity) nor the effects of non-zero recovery rates (absence of recovery). Others focus only on the impact of recovery rates but ignore departures from homogeneity. In what follows we summarize what is known about the behavior of Sullivan types of estimates, the most studied within the literature.

In a number of papers Rogers and colleagues (Rogers, Rogers, and Belanger, 1990; Rogers, Rogers and Belanger, 1989; Rogers, Rogers and Branch, 1989) have correctly pointed out that ignoring the possibility of **recovery** can lead

to a pessimistic bias in the expected durations in the healthy status. They use simple multistate models and estimate increment decrement tables to show the differences between these estimates and those that derive from either Sullivan's method or a less used but equally plausible double decrement table. An important point made by these authors is that the cross sectional prevalence rates do indeed reflect, in one way or another, the presence of recovery (see also Crimmins, Saito and Hayward, 1993). But, it should be understood that even under conditions of stationarity, prevalence rates cannot be an accurate reflection of the duration structure of the population by health status at any age. The precise conditions under which this is so remain to be studied.

A comprehensive assessment of sensitivity of Sullivan type of estimates was carried out by Robine and Mathers (1993). In this exercise the authors are mostly interested in the effects of changing rates on the estimates. They focus on the identification of age patterns of disability, recovery and mortality that are consistent with a given set of observed prevalence rates. The patterns of disability, mortality and recovery thus identified are considered to be realistic in that they produce a given set of prevalence rates. They then use simulations to reproduce various scenarios combining varying recovery and disability rates. Because they do have information on transition and prevalence rates they are able to compare Sullivan type of estimates to those derived from increment-decrement tables. They conclude that for realistic scenarios, "the difference between the estimates produced by the two methods is small and that Sullivan's method is acceptable for monitoring trends in health expectancies for populations", though they acknowledge that this conclusion is more applicable to long term trends that to short term estimation. Finally, it is not clear from this exercise if their conclusions hold at all when there are either constant or varying mortality differentials by health status or when recovery rates change over time.

Another comparison between Sullivan type of estimates and those obtained from the application of multistate methods was carried out by Saito, Crimmins and Hayward (1991; see also Crimmins, Hayward and Saito, 1993). But the conclusions from this study address an issue that is somewhat different from that related to the robustness of Sullivan's types of estimates. Indeed the authors are more interested in showing that calculations using multistate life table methods are sensitive to the instability of observed transition rates in conventional two-wave panel studies, the basic inputs for the associated estimates of expected duration. This is a valid point since a proper comparison between estimates from life table methods and Sullivan procedures must take into account the all sources of variability affecting them, not just departures from assumptions. In the exercise we perform in this paper we take into account stochastic variation and comparisons between one and the other types of estimates are carried out examining the behavior not just of point estimates but also of the standard deviations associated with the increment-decrement procedures. However valid the issue raised by these authors may be, it does not by itself lead to a proper evaluation of deviations from assumptions made when implementing Sullivan's

method.

Finally, in a very thorough simulation exercise Lievre, Brouard and Heathcote (2003) take issue with the value of Sullivan's estimate for verification of theories regarding the compression (expansion) of morbidity. They remark that although Sullivan's estimates can lead to reasonable accurate assessments, they may also lead to misleading conclusions in the absence of stationariy as the momentum of various cohorts with different experiences of mortality, disability and recovery are reflected in cross-sectional prevalence and mortality rates. Theirs is a call for caution when using period measures (such as Sullivan's estimates) when in fact the researchers want to make inferences regarding the evolution over time of disability and mortality. This objection can also be made when comparisons involve various social groups, rather than trajectories over time.

Although the above mentioned studies do indeed confirm the value of cross sectional estimates of life expectancy in disability and health, they also point to the existence of weaknesses. Unfortunately, in none of them there is a systematic evaluation of biases (and corresponding variances) associated with departures from each of the three basic assumptions. For example, although as shown in Appendix 1, homogeneity is required for unbiasedness, in none of the studies is one able to derive an estimate of error when heterogeneity of risks prevails, as is more likely to occur in empirical cases (Lievre, Broaurd and Heathcote, 2003). And what if mortality differentials are combined with the presence of recovery rates? And what if lack of stationarity in mortality rates as well as in mortality differentials act in concert with changing recovery rates?

An important point that deserves consideration is that researchers are not just interested in evaluating expectancies in one place and one point in time but they frequently seek inferences from comparisons across groups at a single or at several points in time. To the extent that departures from key assumptions lead to biases in point estimates, they inevitably infleunce measures of differentials across groups or over time. Depending on the conditions generating the observables, the errors can be smaller, equal or larger than those associated with point estimates. In this paper we do not pursue the study of differentials but do caution that our conclusions may apply with equal or more force to measures of differentials.

3 Methodology

3.1 A simple multistate model

We start with a simple model to describe the trajectories of individual histories under a variety of scenarios. Figure 1 identifies the states and transitions between states that are of interest to us. Individuals aged x pertaining to a cohort born t-x years ago start in the H (healthy) state and may move to an absorbing state (M) via mortality rates ($\mu(x, t-x)$) or to the disabled state (D) via a

disability rate ($\tau(x, t-x)$). A sojourn in D can end either via recovery rates ($\lambda(x, t-x$)) or through mortality rates of disabled individuals ($\varphi(x, t-x)$). This model is not new and was suggested by other researchers concerned about the measurement of disability (Rogers et al., Manton and Stallard, 19, Haberman and Pitacco, 1999; Pollard, Golini and Milella, 1990). The notation we propose makes explicit that rates are functions of age (x) and birth cohort (t-x). Although all rates defined above could also be made dependent of duration of sojourn, we will eschew this complication since it clouds matters and can exert a more pernicious effects. Thus, estimates of errors provided here must be considered as conservative one. To simplify matters and without loss of generality we will assume that all individuals start in H and that the ages of interest are in the range 60-99 years[1].

Figure 1 about here

The assumptions invoked by Sullivan's procedure have three precise implications. First, the assumption of stationariy implies that all rates can be expressed as a function of age (x) only and that one can ignore the birth cohort, namely, $\mu(x, t-x) = \mu(x), \tau(x, t-x) = \tau(x)$, $\varphi(x, t-x) = \varphi(x)$ and $\lambda(x, t-x) = \lambda(x)$. The assumption of homogeneity of risks implies that $\varphi(x) = \mu(x)$.Finally, the assumption of absence of recovery implies that $\lambda(x) = 0$ at all relevant ages. It is possible to show analytically (see Appendix 1) that in the absence of stationarity, the assumptions of homogeneity of risks and absence of recovery are sufficient to produce unbiased estimates of duration in disability with Sullivan's procedure. It can also be shown that the introduction of heterogeneity of risks leads to a bias the magnitude and direction of which is a function dominated by the ratio of mortality risks between healthy and disabled individuals (Palloni, 2004). However, this bias as well as the one generated when the assumption of absence of recovery is removed are not easily calculated except in the simplest case. Their precise magnitude and distributional properties can only be gauged via numerical simulations. The same conclusion applies to the PMM procedure.

As indicated above, the literature on the topic has placed almost exclusive emphasis on violations to stationarity and absence of recovery while deemphasizing the potential consequences of deviations from homogeneity. The latter, however, can be as serious a threat as the other two and lack of robustness of Sullivans as well as the PMM procedure due to lack of homogeneity should not be ignored.

[1] Assuming that all individuals start out in state H at age 60 is a simplification. However, because we simulate a very large number of individual trajectories, the ultimate distribution of the population by health status only depends on the rates, not on the initial distributions.Furthermore, assuming that there is no disability at the first age we consider in the simulations will downplay the errors generated by mortality differentials and the presence of non-zero recovery rates.

3.2 The simulation model: definition of scenarios

We use GENESIS a program designed by Douglas Wolf and written by Jon (Wolf, 1988; Wolg and Jonhson, 1992) to simulate multistate hazard models. We define a number of scenarios to assess errors associated with deviations from the assumptions of homogeneity and absence of recovery or from a combination of the two. In all cases, the scenarios assume stationarity.

Each simulation contains the health trajectories of 10,000 individuals starting at age 60 in the healthy state and ending at age 99. The resulting trajectories are then summarized via calculation of "observed" transition rates, estimation of an increment-decrement table and the corresponding life table functions, including expected years of life, expected years of life in the healthy state, expected years of life in disability and the fraction of all years lived in each state. All functions are evaluated in the age interval 60-99. The trajectory of each cohort can then be described with the simple increment-decrement tables and all its associated functions, including rates and conditional probabilities of moving from one state to another at any age. These are the quantities one would observe if true longitudinal data across several cohorts were available to us. They are used to calculate the "true" values of expected number of years in health and disability.

We then construct tables of counts of transitions and exposure by age and the corresponding total mortality rates and prevalence of disability rates that would be observable if all we had available was cross-sectional information on the population. In essence this amounts to have for each age x, the number of healthy individuals, the number of disabled individuals and, finally, the number of people who died at a particular age. No information is assumed about the initial conditions of individuals who die. Using these observables we implement two procedures to estimate life expectacny with disability.

3.2.1 Sullivan's approach

The first procedure corresponds to the well-known Sullivan calculations that yield marginal expectancies in various states. This method only requires information on the product of the complement of prevalence rates in the age interval x, x+1 and the number of persons years lived in the same interval from a single decrement (overall mortality) life table to allocate years of disability free life expectancy. The resulting partition of the Lx values of the life table can be aggregated to yield estimates of marginal life expectancies in various states. These are calculated as follows:

$$E_x^H = \sum_x L_x(1 - P_x)$$

$$E_x^D = \sum_x L_x P_x$$

where Lx is the estimated number of years lived in the interval (x,x+1) in a life table with mortality rates $\mu(x)$ and P_x isthe observed prevalence rate of disability in the age interval; (x,x+1).

3.2.2 The PMM approach

The second and less known procedure (PMM) uses the observed prevalence rates and the overall mortality rates in a cross section and proceeds to estimate the transition rates from H to D. Its main assumption is that the transition from D to H is zero. The reason we include this procedure in our evaluation is that it too can be valuable since it enables the investigatgor to escape the apparently tight constraint imposed by Sullivan's procedure when assuming no mortality differentials by health status. But it, as does Sullivant's procedure ignores the existence of recovery rates. If the latter were inconsequential but violations to homogeneity carried the day, a shift from Sullivan method to PMM would be called for. But if both are sensitive to the presence of non-zero recovery rates, no such shift would resolve the problem. The nature of PMM is fully described in Appendix 2 but we provide the essential ingredients here.

The new technique's objective is to generate estimates of transition rates from a cross section. In doing so, it removes some of the restrictive assumptions on which Sullivan method is based at the expense of introducing new albeit somewhat weaker ones. In a nutshell, this procedure requires cross-sectional prevalence rates and overall mortality rates by age. Although this is no different from Sullivan's procedure, the utilization of these quantities is since the main goal of the new technique is to derive estimates of all the transition rates rather than to only calculate estimates of life expectancy in disability. The technique starts out from q_x , the overall conditional probability of dying within the interval (x, x+1). It then uses t_x, the equivalent of P_x, the prevalence rate of disability in the interval x,x+1. It is reasonable to assume that (Haberman and Pitacco, 1999) $q_x^{HD} = w_x \frac{q_x^{DD}}{2}$, where q_x^{DD} is the probability of dying within one year for those who are disabled; w_x is the probability of becoming disabled within one year, and q_x^{HD} is the probability of dying within one year and as disabled for those starting in state H. One can also specify the relationship between q_x^{HH} and q_x^{DD} from real data, where q_x^{HH} is the probability of dying in state H for those who start out the one year interval in H. Defining q_x as a function of the previous probabilities and given that qx and $p_x = 1 - q_x$ are known, one can estimate all the necessary transition probabilities.

An increment decrement table representing transitions in the multistate system depicted in Figure 1 cannot be estimated unless one has available information on the rates associated with each transition. This is frequently what is lacking in cross-sectional data. However, one can use the conditional probabilities approximated as suggested above to estimate the associated increment-decrement tables and thus the marginal expectancies in the health and disabled states. To distinguish it from the true increment decrement table associated with each simulated population, we will refer to the one associated with the PMM procedure as the "indirect" increment-decrement table.

Errors associated with each simulation are evaluated comparing estimates produced by Sullivan and the PMM method for each simulated population to the "true" values derived from the associated true increment-decrement table. For each scenario we repeat the simulations 200 times, a sufficiently high number to generate Monte Carlo variation but also small enough to proceed with some speed in a reasonable fast computer. The final result is a set of differences between estimates and true values and their distribution in 200 simulations for each scenarios. The latter are defined below

3.2.3 Scenarios Ia and Ib: Heterogeneity of mortality risks

We use estimates of mortality risks of healthy and disabled individuals from the 1998 and 2000 waves of Health and Retirement Survey (HRS). To calculate these rates we define as disabled any individual experiencing an ADL or IADL and as healthy as any individual with absence of both ADL and IADL. A Gompertz function was fitted to both sets of observed rates and these were used as the baseline mortality pattern in the simulation. The parameters of each function are in Table 2 and the predicted rates are displayed in Figure 2. The observed transitions from the healthy state to disability were also graduated with a Gompertz function whose parameters are displayed in Table 2. Figure 2 displays the corresponding fitted values. All the equations fit relatively well as the values of R^2 exceed .80 for mortality rates of healthy and disabled individuals as well as for disability rates.

The baseline scenario is one where we force the equality $\varphi(x) = \mu(x)$. **Scenario Ia** is one where the mortality differential is mild (the ratio of the Gompertz level parameters of $\varphi(x)$ relative to $\mu(x)$ is about 2.3 at age 75) and **Scenario Ib** is one where the mortality differential is large (the ratio of the Gompertz parameters is 6.9 at age 75).

3.2.4 Scenarios IIa and IIb: recovery rates are non-zero

The next scenario allows recovery rates to be non zero. Baseline rates for recovery rates were estimated from the 1998 and 2000 waves of HRS. As was the case for disability rates, the observed values were graduated with a Gompertz function whose parameters are also displayed in Table 2. Figure 2 displays the fitted values. The equation for recovery fits less well than the other ones as the R^2 is of the order of 0.63. **Scenario IIa** was defined as one where $\varphi(x) = \mu(x)$ and where the recovery rates are identical to those in Figure 2. **Scenario IIb** was generated with the same recovery rates and the large mortality differential.

Table 2 about here
Figure 2 about here

3.3 Measurement of errors

In order to assess the magnitude and direction of biases, we use two types of errors, the squared root of the quadratic error and the absolute values of the relative error. All errors are calculated using estimates and true parameters *at each age within the range considered here (60-99).* Thus, for each simulated population we obtain 40 values for the quadratic errors and 40 for the absolute value of the relative error. The distribution of errors for a given age over the range of simulations will thus contain 200 observations. Each one of these distributions is characterized by the its mean value and its standard deviation. We take the mean as a measure of the bias inherent in the procedures (Sullivan's or PMM method) while the standard deviation is an index of stochastic variability. A large coefficient of variation associated with a given scenario suggests that biases are small relative to stochastic variation whereas a small value of the coefficient of variation indicates the opposite pattern. To simplify presentation we will only discuss the relative errors. Examination of quadratic errors leads to very similar conclusions.

4 Results

In what follows we evaluate the absolute values of relative errrors (AVRE) as well as their standard deviations (over the set of 200 simulations) for each scenario. The main results for selected ages are presented in Tables 3a to 3e.

Tables 3a-3e about here

For the PMM procedure we produce two sets of results: the first set is obtained under the assumption that there are no mortality differentials between healthy and disabled populations (denoted as PMM1) whereas the second assumes the existence of a differential equal to the one that generates simulated populations with a mild mortality differential (denoted as PMM2). We calculate these to investigate *whether or not under ignorance about the magnitude of heterogeneity of risks there is a pay off in trying to guess it or, alternatively, whether the errors induced by incorrectly guessing the size of differentials are larger than if one would have simply used the simpler Sullivan procedure.*

Figure 3 displays the errors (figures on the left) and standard deviations (figures on the right) *by method* and for each one of five scenarios described above. The graph enables us to compare magnitude and direction of errors across methods within a particular scenario. Figure 4, on the other hand, displays errors in a slightly different way: each graph shows the patterns of error for a single method thus allowing us to judge the magnitude and direction of errors across scenarios for a given method. Because the magnitude of relative errors and the behavior of the methods are very different at very old ages (over

90), we choose to display separately some figures for two broad age intervals, 60-90 and over 90.

Finally, Tables 4a to 4e display the coefficients of variation for selected ages obtained from each method and within each simulated scenario.

Tables 4a-4e about here
Figures 3, 4 and 5 about here

4.1 Comparison of methods

As expected, we find that in the baseline scenario (Figure 3, row 1, no mortality differences and no recovery) the magnitude of errors is small for both procedures provided that the PMM method is applied assuming no mortality differences (PMM1). The errors associated with Sullivan procedures are slightly higher but never exceed 1 percent and are well within the bounds of stochastic variability. The errors increase quite sharply above age 95 and this is certainly due to the fact that calculated rates from simulated data become very unstable at these ages as the population exposed to events has thinned out considerably. The increase in these errors should be taken as a simple warning suggesting that calcualtions of estimates at these ages is failry hazardous even if assumptions are met rigorously. In **scenario Ia** (Figure 3, row 2, low mortality differences between disabled and non disabled people and no recovery), the errors are also small (close to one per cent) for both PMM2 (Pseudo Multistate Model calculated assuming mild heterogeneity in mortality risks) and Sullivan's procedure. As in the baseline scenario, errors at very old ages are quite large but more so for Sullivan procedure than for PMM2.

When mortality rates for disabled people are significantly higher than mortality rates for healthy individuals as it happens in **Scenario Ib** (Figure 3, third row) the error for both the Sullivan's methods and PMM2 increases significantly and can reach values as high as 8 to 10 percent for Sullivan procedure and as high as 15 percent for PMM2. Paradoxically the PMM2 procedure–which led to estimates that assume differential mortality-produces errors that are worse than those generated by the simpler procedure suggested by Sullivan. The values for PMM2 were calculated using a mild mortality differentials not a high one. This shows that if the assumption about mortality differentials made in the implementation of the PMM2 procedure is off target, it will lead to worse errors than if one ignored altogether the existence of mortality differentials.

Scenario IIa is the first scenario with recovery (Figure 3, row 4). In it we assume no mortality differences and therefore, the errors associated with the procedures are solely attributable to violation of the assumption about absence of recovery. The errors appear to decrease with age, a possible consequence of the fact that recovery rates too decrease sharply with age and are never larger than 5 percent in all cases. Estimates of the PMM1 type lead to errors that are larger than those associated with Sullivan procedure but not by much and the differences are well within the bounds of stochastic variability. Thus, for

example, whereas the average error at age 60 is 3.4% for Sullivan's method, it is about 4.8% for PMM1 (Table 3d). However, the standard deviation of the errors is relatively sizeable and a simple test of statistical significance would lead to a rejection of the hypotheses that the two procedures produce significantly different results.

Assuming the existence of both recovery and large mortality differences, as is done in **Scenario IIb** (Figure 3, row 5) leads to somewhat different patterns. First, the maximum magnitude of errors grows somewhat in size as values can grow to be close to 10 or 11 percent. Second, a comparison of Sullivan's method with the PMM2 procedure suggests, here again, that the latter yields much larger errors. Furthermore, Sullivan's procedure produces errors of lower magnitude than uder **Scenario IIa**. Although this may seem paradoxical, it can be explained by the fact that the existence of recovery should attenuate the importance of mortality differentials as individuals are, one average, exposed to both mortality regimes.

Two remarks are important. First, in almost all cases the behavior of the standard deviations of errors mirrors closely the behavior of the errors themselves, in age patterns as well as in magnitude. Indeed, as shown in Tables 4a-4e the coefficients of variation associated with each scenario and method are centered around 1. This suggests that errors induced by deviations from assumptions are of magnitude similar to the variance produced by Monte Carlo variation. Second, in general and up to age 95, the errors in the estimation of life expectancy with disability when using Sullivan's method are small (close to one per cent absolute relative error). Only for the simulated scenario which assumes recovery (**Scenario IIa**) for younger ages, are the errors associated with Sullivan's method higher (about 3.4% at age 60). In general, in s**cenarios IIa and IIb** as age increases the errors decrease as well, mirroring the decreasing pattern of recovery rates. But at older ages (over 95) Sullivan's method produces very large errors (see Figure 5).

4.2 Comparison of scenarios

We now examine the behavior of errors associated with each method by type of scenario. The two graphs in the first row of Figure 4 display the errors associated with Sullivan type of estimates in each of the scenarios we simulated. The plot on the left compares scenarios that assume mortality differentials (Ia and Ib) with the baseline scenario. The plot on the right compares scenarios that assume the existence of positive recovery rates from disability to the healthy state (IIa and IIb) with the baseline scenario. In the second and third rows, we show the same comparisons for PMM1 and PMM2, respectively.

The results show that in all cases errors in the estimation of life expectacy with disability using the Sullivan method are small, at least until age 95. The errors are higher when heterogeneity of risks is large (**Scenarios Ia and Ib**). For example, at age 80, when we assume mild mortality differentials and no recovery (Ia) the average error (AVRE) for Sullivan's method is about 1.3% (see

Table 3b). On the other hand, when we the simulated population experiences involves a large mortality gap between healthy and disabled (Ib), the average error is twice as large (2.7%, see Table 3c). These are modest errors but only apply to ages below 95. Beyond that age errors grow above 10 percent

Figure 5 shows the performance of Sullivan's method in two broad age groups, namely 60 to 90 years of age and 91 years and above. For the first age group the errors associated with Sullivan types of estimates are lower in a scenario with recovery and when risks are heterogeneous (large mortality differentials). As mentioned before, a plausible explanation for the robustness of Sullivan's estimates in this case is that the presence recovery compensates for the higher mortality gap: the higher the recovery rate is the higher the proportion of individuals that die in the healthy state, and therefore the lower the effect of a mortality differentials between healthy and disabled individuals. Nevertheles, this compensation effect is influenced by the magnitude of the disability rates. To the extent that disability rates are high there would still be a large number of individuals who experience disability, even with a large recovery rate. As a consequence, the effect of over mortality among disabled individuals will have a larger influence on the error of the estimates (which do not account for this mortality gap). Given that the largest recovery rates are located in the age groups with lowest disability rates, we should expect stronger offsetting of errors in the younger ages (60 to 90 years), as it is the from the results displayed in Figure 5.

For the older age groups, (aged 91 years and above) the offsetting effects weaken and the errors grow much larger under **Scenario IIb**. This results is consistent with the previous reasoning since it is these older groups that experience lower recovery rates so that the effect of a large mortality gap between disabled and healthy individuals becomes dominant precisely among them.

Figure 4, row 2, displays the behavior of errors associated with PMM1. For this method, the **Scenario Ia** implies low errors in the estimation of life expectancy with disability. An important point is that this precision is maintained even at older ages where Sullivan's method begins to produce poor results.Under the more realistic scenarios (IIa and IIb) including recovery, errors are about 6% for younger ages but performance deteriorates at older ages.

Errors associated with PMM2 are displayed in the third row of Figure 4. The estimates behave in a similar way as do estimates obtained via PMM1, but the magnitude of errors is higher when the mortality gap is larger than the gap assumed by PMM2. Clearly, PMM2 is not as robust as Sullivan's estimates to underestimation of mortality differentials between disabled and healthy individuals. It seems ill-advised to use PMM2 unless one is certain to capture well the true value of mortality heterogeneity.

5 Summary and conclusions

We study the performance of Sullivan's method and that of a new approach to estimate life expectancy with disability. The new approach differs from Sul-

livan's in that it relies on an approximation of transition probabilities, rather than on the direct estimation of life expectancy with disability. We evaluate relative errors under violations of two of the three crucial assumptions on which these procedures rely: absence of recovery and homogeneity of risks. The main conclusion seems to be that the magnitude of errors is modest, even under the worse case scenario of departure from the basic assumption regarding homogeneity of risks. Errors are large, but still within tolerable levels, in populations with non zero rates of recovery. However, both methods tend to perform badly at very old ages though part of the poor performance is a matter of stochastic variability due to instability associated with low number of observations. Specially remarkable is the fact that Sullivan's method performs well, even in situations that combine heterogenity of risks and non-zero recovery rates. Naturally, errors grow larger when the magnitude of the mortality differentials increase but one would need very sizeable heterogenity of risks to produce errors exceeding 10 percent. The PMM procedure performs as well as Sullivan's method but only when the underlying mortality differential is guessed correctly. Otherwise, its robustness breask down. This seems to indicate that under ignorance regarding the precise magnitude of mortality differentials, one should always prefer Sullivan procedure. Of special notice is the fact that the presence of recovery rates tends to offset the errors induced by heterogeneity of mortality risks, the more so in ages where recovery tends to be of higher intensity.

Not all of this is good news, however. First, it should be remembered that we ignore throughout violations to the assumptions of stationarity, an assumptions that other researchers have thought could lead incorrect estimates. Undoubtedly lack of stationarity in mortality, recovery and disability rates may combine to expand the magnitude of errors shown above. Of particular relevance is the issue of non stationarity in recovery rates, a phenomenon likely to occur under regimes of rapidly changing medical technology. Second, we did not consider at all the existence of attrition due to causes other than mortality. One consequential source of attrition is institutionalization of the elderly, a mechanism through which individuals are removed from the observed population. To the extent that this occurs selectively by health status, it will only compound the errors calculated before. Finally, Sullivan estimates have been used extensively to compare different subgroups or even to compare the conditons in different periods of time. Both types of contrast mat be grossly in error if, for example, conditions are violated to different extent in subgroups or within certain periods of time. Thus, estimates of differential life expectacies in disability could be very fragile, more so than those associated with a population as a whole or with a single period of time.

This simulation exercise can be summarized with a few prescriptions that only hold under stationarity. First, by an large Sullivan' estimates perform well even under gross violations of assumptions but not always at older ages. Second, the PMM method also performs well when there is risk heterogeneity provided that the initial guess about mortality differentials between disabled and healthy

is correct. Otherwise, it leads to errors that can be larger than those associated with Sullivan's procedure. However, it should be remembered that the PMM procedure yields quantities other than expected durations in disability. Yet in this paper we have not evaluated errors in estimated transitions rates for the PMM procedure and thus we cannot make statements about its desirability even when target quantitites are not expected durations only. Third, large mortality differentials between disabled and healthy can and do produce relatively sizeable errors with either procedure. And so do the existence of non-zero recovery rates. Yet, the combination of these and large mortality differentials generate offsetting effects that prevent the occurrence of worse errors than under each scenario separately.

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Table 1: Summary of Methodologies to Estimate Expectancies in Health and Disability

Method	Assumptions	Data Required
Sullivan	Homogeneity of risks Stationarity Absence of recovery	Cross Sectional Mortality Rates Cross Sectional Prevalence Rates
PMM	Stationarity Absence of recovery Known mortality differentials	Cross Sectional Mortality Rates Cross Sectional Prevalence Rates
Increment-Decrement (I)	Stationarity	Limited (Two waves) Panel Data
Increment-Decrement II	————	Full Longitudinal Information

TABLE 2[1]
Estimation results for the Gompertz Models
Age over 60 years

	Healthy Mortality rate	Disabled Mortality rate	Rate of becoming disabled	Recovery rate
Intercept	1.5900E-05**	0.0004**	0.0006*	2.4603**
	(7.3974E-06)	(0.0019)	(0.0001)	(0.9865)
b_1	0.0981*	0.0656*	0.0650*	-0.0395*
	(0.0059)	(0.0055)	(0.0030)	(0.0051)
R Square	0.8872	0.7952	0.9280	0.6325
F (1,35)	275.4339	-	-	60.2407
F (1,36)	-	139.7722	463.9160	-
P Value	0.0000	0.0000	0.0000	0.0000

* Statistically significant at the one percent level (two tail test)

** Statistically significant at the five percent level (two tail test)

(1) All regression were fitted to data for ages from 60 to 99 and the form of all the regressions was ln y(x) =a+bx where y(x)is the rate at age x and x is the age. The parameter "intercept" above is exp(a) and is the parameter level of the Gompertz whereas the parameter b_1 is the slope of the Gompertz curve

TABLE 3a
Absolute Value of Relative Error
In Expected Duration of Disability and Standard Deviation
Baseline Scenario: No Differences Mortality and No Recovery

Age	AVRE_SM	SE_SM	AVRE_PMM1	SE_PMM1	AVRE_PMM2	SE_PMM2
60	0.009308	0.009221	0.005141	0.005113	0.007561	0.007504
65	0.009527	0.009436	0.005272	0.005243	0.007569	0.007511
70	0.010018	0.009917	0.006520	0.006477	0.008494	0.008422
75	0.010650	0.010537	0.008239	0.008171	0.009804	0.009708
80	0.011068	0.010945	0.009659	0.009565	0.010820	0.010703
85	0.010678	0.010564	0.009780	0.009684	0.010644	0.010530
90	0.009166	0.009081	0.008063	0.007998	0.008878	0.008799
95	0.008292	0.008202	0.005106	0.005079	0.007107	0.007045

AVRE: Absolute Value of Relative Error
SE: Standard Error
SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

TABLE 3b
Absolute Value of Relative Error
In Expected Duration of Disability and Standard Deviation
Ia Scenario: Low Mortality Differences and No Recovery

Age	AVRE_SM	SE_SM	AVRE_PMM1	SE_PMM1	AVRE_PMM2	SE_PMM2
60	0.008765	0.008689	0.007245	0.007190	0.010208	0.010103
65	0.009063	0.008981	0.007616	0.007557	0.010588	0.010475
70	0.009853	0.009756	0.009146	0.009062	0.012060	0.011914
75	0.011069	0.010947	0.011194	0.011068	0.013913	0.013719
80	0.012546	0.012388	0.013180	0.013006	0.015512	0.015271
85	0.013682	0.013494	0.014144	0.013943	0.015946	0.015691
90	0.013632	0.013443	0.013041	0.012870	0.014461	0.014250
95	0.013060	0.012860	0.009425	0.009334	0.011993	0.011834

AVRE: Absolute Value of Relative Error
SE: Standard Error
SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

TABLE 3c
Absolute Value of Relative Error
In Expected Duration of Disability and Standard Deviation
Ib Scenario: High Mortality Differences and No Recovery

Age	AVRE_SM	SE_SM	AVRE_PMM1	SE_PMM1	AVRE_PMM2	SE_PMM2
60	0.011253	0.011127	0.028947	0.028099	0.036361	0.035035
65	0.012179	0.012031	0.030120	0.029207	0.038189	0.036728
70	0.014621	0.014407	0.033631	0.032496	0.043611	0.041708
75	0.019114	0.018748	0.039054	0.037525	0.052383	0.049637
80	0.026983	0.026255	0.046956	0.044746	0.065697	0.061378
85	0.040321	0.038692	0.056961	0.053695	0.085553	0.078226
90	0.062861	0.058891	0.062791	0.058655	0.114764	0.101564
95	0.099321	0.089231	0.058058	0.053886	0.151656	0.128412

AVRE: Absolute Value of Relative Error
SE: Standard Error
SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

TABLE 3d
Absolute Value of Relative Error
In Expected Duration of Disability and Standard Deviation
IIa Scenario: No Mortality Differences and Recovery

Age	AVRE_SM	SE_SM	AVRE_PMM1	SE_PMM1	AVRE_PMM2	SE_PMM2
60	0.034236	0.033063	0.048141	0.045816	0.049200	0.046776
65	0.031472	0.030481	0.042713	0.040885	0.044286	0.042323
70	0.026166	0.025481	0.034314	0.033134	0.036882	0.035521
75	0.020522	0.020100	0.026608	0.025899	0.030258	0.029341
80	0.015318	0.015083	0.020536	0.020113	0.025051	0.024423
85	0.011351	0.011222	0.016313	0.016046	0.021366	0.020908
90	0.009150	0.009064	0.013154	0.012979	0.018715	0.018362
95	0.009199	0.009085	0.008478	0.008392	0.017481	0.017154

AVRE: Absolute Value of Relative Error
SE: Standard Error
SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

TABLE 3e
Absolute Value of Relative Error
In Expected Duration of Disability and Standard Deviation
IIb Scenario: High Mortality Differences and Recovery

Age	AVRE_SM	SE_SM	AVRE_PMM1	SE_PMM1	AVRE_PMM2	SE_PMM2
60	0.017181	0.016885	0.061088	0.057334	0.061722	0.057903
65	0.011607	0.011472	0.051399	0.048742	0.052918	0.050113
70	0.003461	0.003448	0.038531	0.037035	0.042799	0.040964
75	0.002914	0.002905	0.028955	0.028108	0.037933	0.036492
80	0.005766	0.005731	0.024422	0.023816	0.040207	0.038588
85	0.002737	0.002726	0.026256	0.025547	0.051701	0.049023
90	0.010609	0.010473	0.032595	0.031416	0.074392	0.068835
95	0.040548	0.038622	0.042569	0.040232	0.111271	0.098610

AVRE: Absolute Value of Relative Error
SE: Standard Error
SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

TABLE 4a
Coefficient of Variation
Baseline Scenario: No Differences Mortality and No Recovery

Age	SM	PMM1	PMM2
60	0.9907	0.9946	0.9923
65	0.9905	0.9946	0.9924
70	0.9900	0.9934	0.9915
75	0.9893	0.9917	0.9902
80	0.9889	0.9903	0.9892
85	0.9893	0.9902	0.9893
90	0.9907	0.9919	0.9910
95	0.9891	0.9947	0.9914

SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

TABLE 4b
Coefficient of Variation
Ia Scenario: Low Mortality Differences and No Recovery

Age	SM	PMM1	PMM2
60	0.9912	0.9924	2.6040
65	0.9909	0.9922	2.4206
70	0.9901	0.9908	4.5662
75	0.9889	0.9888	3.9022
80	0.9874	0.9868	0.7850
85	0.9863	0.9858	0.2721
90	0.9862	0.9869	0.0930
95	0.9847	0.9904	0.0273

SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

TABLE 4c
Coefficient of Variation
Ib Scenario: High Mortality Differences and No Recovery

Edad	SM	PMM1	PMM2
60	0.9887	0.9707	0.9635
65	0.9878	0.9697	0.9618
70	0.9854	0.9663	0.9564
75	0.9809	0.9609	0.9476
80	0.9730	0.9529	0.9343
85	0.9596	0.9427	0.9144
90	0.9369	0.9341	0.8850
95	0.8984	0.9282	0.8467

SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

TABLE 4d
Coefficient of Variation
IIa Scenario: No Mortality Differences and Recovery

Edad	SM	PMM1	PMM2
60	0.9657	0.9517	0.9507
65	0.9685	0.9572	0.9557
70	0.9738	0.9656	0.9631
75	0.9795	0.9733	0.9697
80	0.9847	0.9794	0.9749
85	0.9886	0.9836	0.9786
90	0.9906	0.9867	0.9812
95	0.9876	0.9898	0.9813

SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

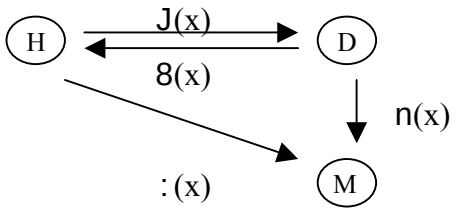
TABLE 4e
Coefficient of Variation
IIb Scenario: High Mortality Differences and Recovery

Edad	SM	PMM1	PMM2
60	0.9828	0.9385	0.9381
65	0.9883	0.9483	0.9470
70	0.9964	0.9612	0.9571
75	0.9968	0.9707	0.9620
80	0.9940	0.9752	0.9597
85	0.9959	0.9730	0.9482
90	0.9872	0.9638	0.9253
95	0.9525	0.9451	0.8862

SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

FIGURE 1

STATES AND TRANSITIONS IN A MULTIPLE STATE MODEL OF DISABILITY



H: HEALTHY; D: DISABLED; M: DEAD

FIGURE 2
Observed and fitted values for the Gompertz Models

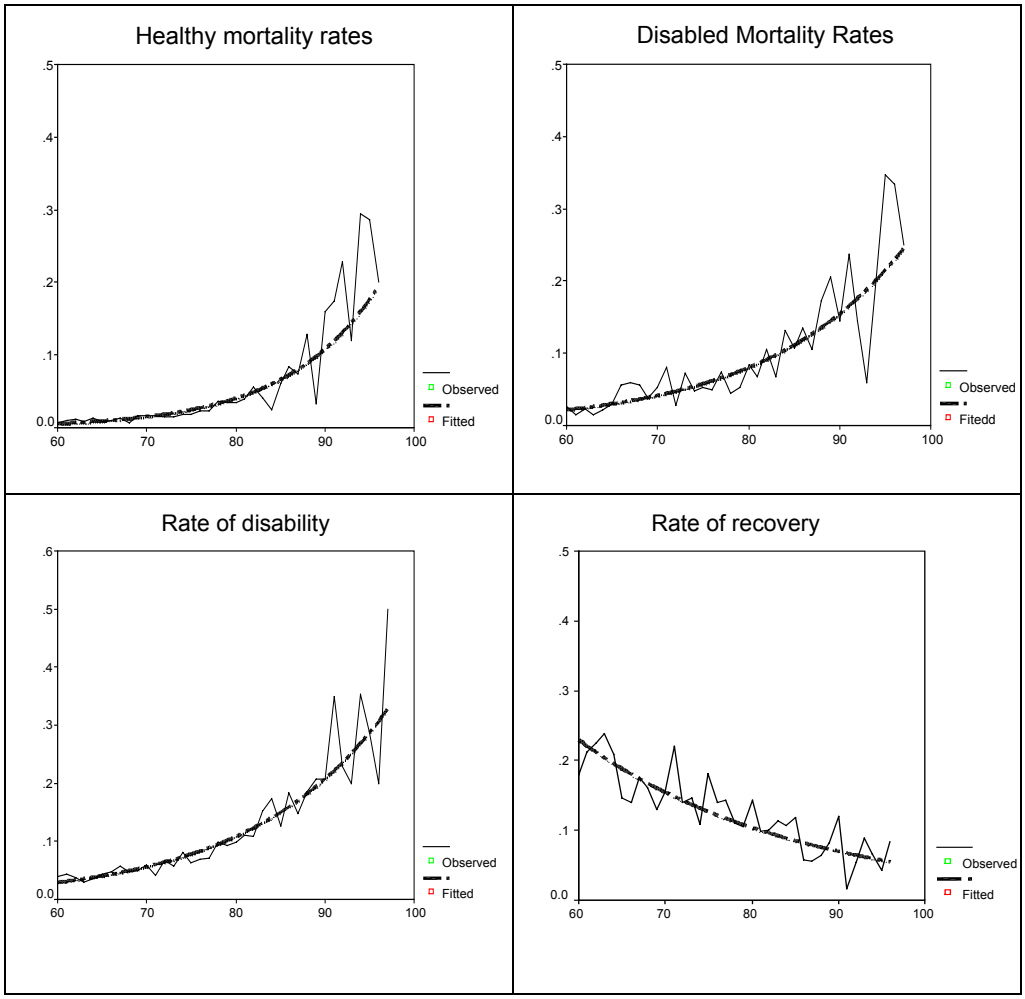
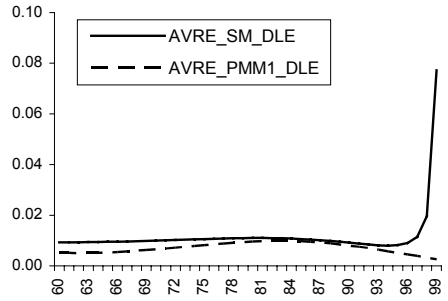


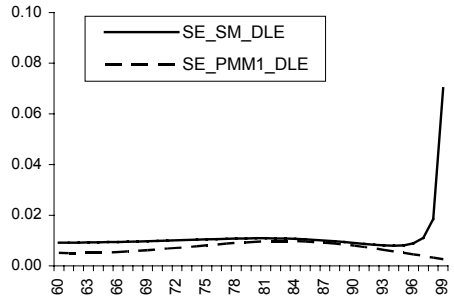
FIGURE 3
Absolute Value of Relative Error (AVRE) in Expected Duration of Disability and Standard Error (SE)
Comparisons of Methods

Baseline Scenario



Absolute Value of Relative Error

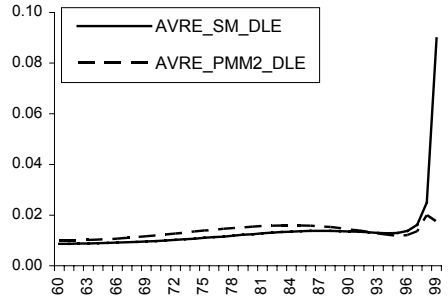
Sullivan'n Method and Pseudo Multistate Model



Standard Deviation AVRE

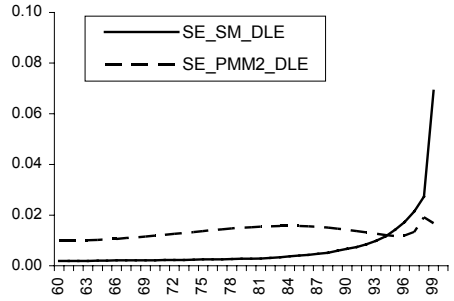
Sullivan'n Method and Pseudo Multistate Model

Ia Scenario



Absolute Value of Relative Error

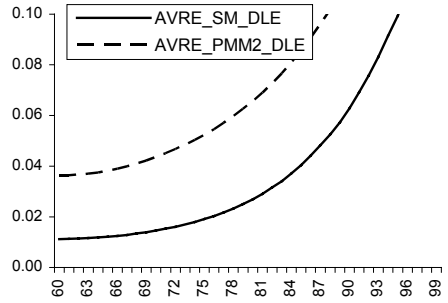
Sullivan'n Method and Pseudo Multistate Model



Standard Deviation AVRE

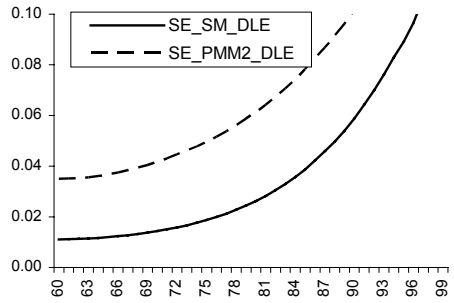
Sullivan'n Method and Pseudo Multistate Model

Ib Scenario



Absolute Value of Relative Error

Sullivan'n Method and Pseudo Multistate Model



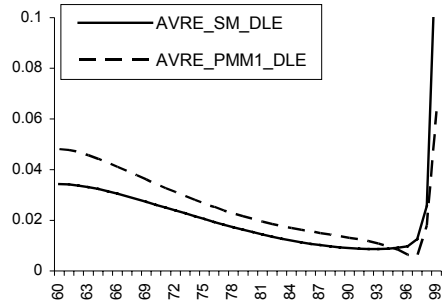
Standard Deviation AVRE

Sullivan'n Method and Pseudo Multistate Model

DLE: Disabled life expectancy.
SM: Sullivan's Method; PMM1: Pseudo Multistate Model (no mortality differences); PMM2: Pseudo Multistate Model (mortality differences).

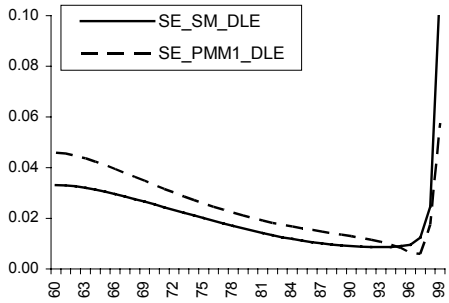
FIGURE 3 (cont.)
Absolute Value of Relative Error (AVRE) in Expected Duration of Disability and Standard Error (SE)
Comparisons of Methods

IIa Scenario



Absolute Value of Relative Error

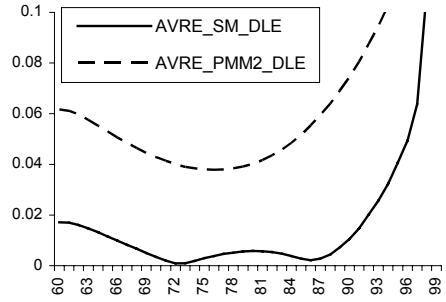
Sullivan'n Method and Pseudo Multistate Model



Standard Deviation AVRE

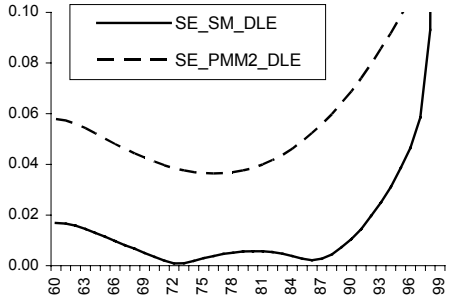
Sullivan'n Method and Pseudo Multistate Model

IIb Scenario



Absolute Value of Relative Error

Sullivan'n Method and Pseudo Multistate Model



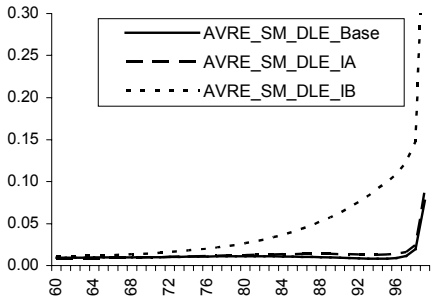
Standard Deviation AVRE

Sullivan'n Method and Pseudo Multistate Model

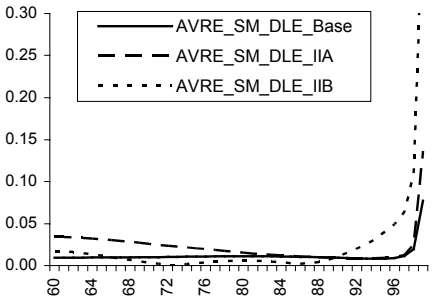
DLE: Disabled life expectancy
SM: Sullivan's Method; PMM1: Pseudo Multistate Model (no mortality differences); PMM2: Pseudo Multistate Model (mortality differences).

FIGURE 4
Absolute Value of Relative Error
In Expected Duration of Disability
Comparisons of Scenarios

Sullivan'n Method

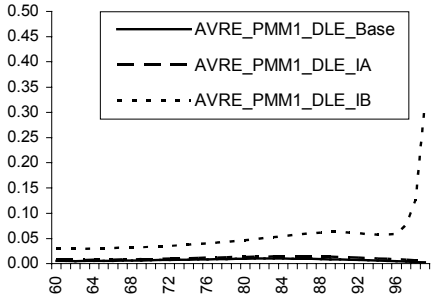


Scenarios: Baseline, Ia and Ib

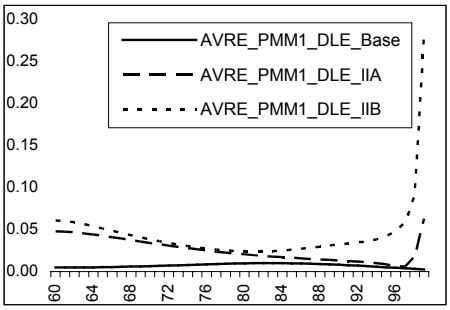


Scenarios: Baseline, IIa and IIb

Pseudo Multistate Model (1)

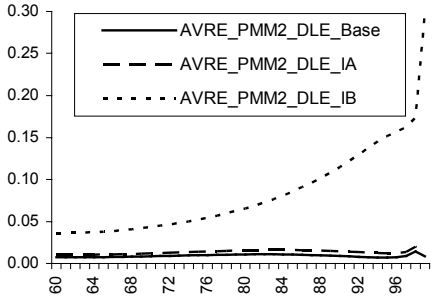


Scenarios: Baseline, Ia and Ib

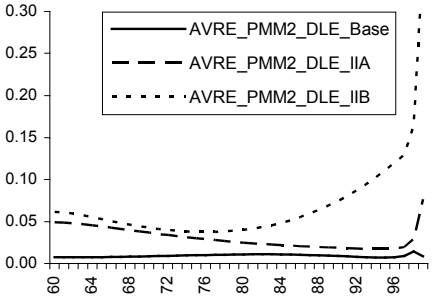


Scenarios: Baseline, IIa and IIb

Pseudo Multistate Model (2)



Scenarios: Baseline, Ia and Ib

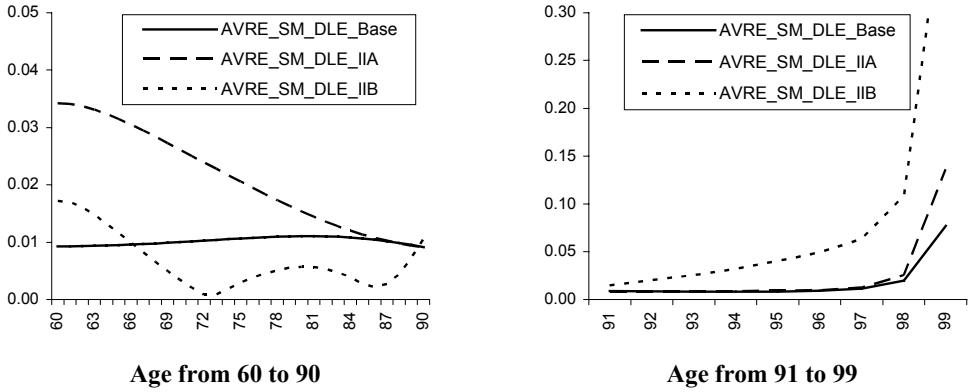


Scenarios: Baseline, IIa and IIb

DLE: Disabled life expectancy
SM: Sullivan's Method
PMM1: Pseudo Multistate Model (no mortality differences)
PMM2: Pseudo Multistate Model (mortality differences)

FIGURE 5
Absolute Value of Relative Error
In Expected Duration of Disability
Scenarios: Baseline, IIa and IIb

Sullivan'n Method



Age from 60 to 90 **Age from 91 to 99**

DLE: Disabled life expectancy
SM: Sullivan's Method

Appendix 1

In what follows we show that if the assumtpion of stationary and absence of recovery prevail, a sufficient condition for Sullivan's estimates to be unbiased is that the force of mortality among diasbled be identical to that of healthy individuals (homogeneity of risks).

Let $u(x)$ be the force of mortality at x for healthy, $u^*(x)$ be the force of mortality at x for disabled and $r(x)$ be the force of disability at x.

The number of persons years lived as disabled is given by:

$$L_x^D = \int_x^{x+1} \int_0^z \exp\left\{-\int_0^y [u(v) + r(v)]dv\right\} r(y) \exp\left[-\int_y^z u^*(v)dv\right] dydz.$$

Assume that L_x (number of persons years lived in total) can be evaluated within $(x, x+1)$ at point z^*, $x \leq z^* < x+1$,

$$L_x^D = \int_0^{z^*} \exp\left\{-\int_0^y [u(v) + r(v)]dv\right\} r(y) \exp\left[-\int_y^{z^*} u^*(v)dv\right] dy.$$

If $u(x) = u^*(x)$ we have,

$$\begin{aligned} L_x^D &= \int_0^{z^*} \exp\left[-\int_0^{z^*} u(v)dv\right] r(y) \exp\left[-\int_0^y r(v)dv\right] dy \\ &= \exp\left[-\int_0^{z^*} u(v)dv\right] \int_0^{z^*} r(y) \exp\left[-\int_0^y r(v)dv\right] dy \\ &= L_{z^*} Q_{z^*}^D, \end{aligned}$$

where L_{z^*} is the number of persons years lived in total life table at z^* and $Q_{z^*}^D$ is the probability of becoming disabled by age z^* in the absence of mortality (single decrement table).

To complete the demonstration we only need to show that when $u(x) = u^*(x)$ the observed prevalence of disability at any age z^* $x < z^* <= x+1$, namely $P(z^*)$, equals $Q_{z^*}^D$.

The prevalence of disability in age z^* is:

$$P(z^*) = \frac{N(z^*)}{\overline{N}(z^*) + N(z^*)},$$

where $N(z^*)$ is the number of disabled in age z^* and $\overline{N}(z^*)$ is the number of non-disabled in age z^*.

$$P(z^*) = \frac{\int_0^{z^*} \exp\left\{-\int_0^y [u(v) + r(v)]dv\right\} r(y) \exp\left[-\int_y^{z^*} u^*(v)dv\right] dy}{\exp\left\{-\int_0^{z^*} [u(v) + r(v)]dv\right\} + N(z^*)}$$

where $N(z^*)$ is the numerator of $P(z^*)$.

$$P(z^*) = \frac{\exp\left[-\int_0^{z^*} u(v)dv\right]\int_0^{z^*} \exp\left[-\int_0^{y} r(v)dv\right]r(y)dy}{\exp\left\{-\int_0^{z^*}[u(v)+r(v)]dv\right\} + \exp\left[-\int_0^{z^*} u(v)dv\right]\int_0^{z^*} r(y)\exp\left[-\int_0^{y} r(v)dv\right]dy}$$

$$= \frac{\int_0^{z^*} \exp\left[-\int_0^{y} r(v)dv\right]r(y)dy}{\exp\left[-\int_0^{z^*} r(v)dv\right] + \int_0^{z^*} r(y)\exp\left[-\int_0^{y} r(v)dv\right]dy}$$

The denominator of the previous expression equals 1 in the single decrement table where only disability operates as a decrement. Thus

$$P(z^*) = \int_0^{z^*} \exp\left[-\int_0^{y} r(v)dv\right]r(y)dy = Q^D_{z^*}.$$

This proves that when there is ***no recovery, when stationarity prevails and when*** $u(x) = u^*(x)$ (homogeneity of risks), Sullivan's calculations based on the producst $L^D_x = L_x \cdot P(x)$ are correct. Thus, homogeneity of risks is a sufficient condition for the robustness of Sullivan's type of estimates .

Appendix 2. The Pseudo Multistate Model

We denote by q_x the annual probability of death for an individual of age x, from any initial state. We will suppose that this probability is fixed and known. We will take the raw mortality rates provided by the HRS. We denote by t_x the probability of being disabled estimated from the disability prevalence rate at age x, which will be obtained from the same survey.

The probability of a person aged x dying before reaching age $x+1$ can be calculated:

$$q_x = (1-t_x)q_x^{HH} + (1-t_x)q_x^{HD} + t_x q_x^{DD}, \qquad (1)$$

with q_x^{HH} the probability of dying within one year, the death occurring in state H; q_x^{HD} and q_x^{DD} the probabilities of dying within one year, the death occurring in state D.

In the same way, p_x (probability of a person aged x surviving up to age $x+1$) can be calculated as:

$$p_x = (1-t_x)p_x^{HH} + (1-t_x)p_x^{HD} + t_x p_x^{DD}. \qquad (2)$$

with p_x^{HH} the probability of keeping in good health at age $x+1$, p_x^{HD} and p_x^{DD} the probabilities of surviving as disabled at age $x+1$.

The fundamental relations between the previous probabilities can be found in Haberman and Pitacco (1999, p. 96).

Following (1) some assumptions can be made to approximate the transition probability of dying before reaching age $x+1$ for a healthy person of age x (death occurring in the disability state), q_x^{HD}. In a similar way, we can establish some hypothesis to calculate the probability of dying before reaching age $x+1$ for a healthy individual of age x (death occurring in the healthy state), q_x^{HH}. In this sense, two assumptions are defined:

- *Assumption 1.1*

$q_x^{HD} = k_1 w_x q_x^{DD}$; *with* $0 < k_1 \leq 1$, and w_x the probability of becoming disabled within one year ($w_x = p_x^{HD} + q_x^{HD}$).

- *Assumption 1.2*

$q_x^{HH} = b_0 e^{b_1 x} q_x^{DD}$.

In both cases, we use the probability of death for a disabled individual of age x, q_x^{DD}, and the probability of becoming disabled between x and $x+1$, w_x to calculate q_x^{HD} and q_x^{HH}, respectively. In Haberman and Pitacco (1999), $q_x^{HD} = w_x \frac{q_x^{DD}}{2}$.

Assumption 1.1 can be justified in terms of the distribution of the age at disability onset. On the particular case, the factor ½ follows from the assumption of a uniform distribution of age at onset in the interval $(x,x+1)$. Assumption 1.2 has been established to set a relationship between the two probabilities, q_x^{HH} and q_x^{DD}, as it is derived from the HRS results. It is generally accepted that the probability of dying is greater for a disabled than for a healthy person. An exponential function for q_x^{HH} / q_x^{DD} is specified.

The number of disabled people in age $x+1$ is equal to the number of disabled people in x that reach age $x+1$ plus the number of healthy people in x that survive to $x+1$ in the disability state. Therefore,

$$(1-q_x)t_{x+1} = t_x(1-q_x^{DD}) + (1-t_x)(w_x - q_x^{HD}). \quad (3)$$

By means of substituting Assumption 1.1 in expression (3) we obtain the probability of becoming disabled between *x* and *x+1* as:

$$w_x = \frac{t_{x+1}(1-q_x) - t_x(1-q_x^{DD})}{(1-t_x)(1-q_x^{DD}k_1)}. \quad (4)$$

On the basis of Assumptions 1.1 and 1.2, and the expression (1), we obtain the probability that a disabled individual of age *x* dies before reaching age *x+1* as:

$$q_x^{ii} = \frac{q_x}{(1-t_x)(b_0 e^{b_1 x}) + (1-t_x)w_x k_1 + t_x} \quad (5)$$

From (4) and (5), for fixed values of k_1, fitted values of b_0 and b_1, and given q_x and t_x, we can obtain q_x^{DD} and w_x. Firstly, we need to solve the following equation:

$$\left[k_1(t_x - 1)b_0 e^{b_1 x}\right](q_x^{DD})^2 + \left[k_1(t_{x+1} - t_x + q_x(1-t_{x+1})) - b_0 e^{b_1 x}(t_x - 1) + t_x\right](q_x^{DD}) - q_x = 0 \quad (6)$$

Once q_x^{DD} is obtained, q_x^{HD} and q_x^{HH} follow from the equalities in hypothesis 1.1 and 1.2. It can be shown that if expression (6) has two real solutions, both of them are positives, but only one of them will always be in the interval (0,1).

Finally, we obtain the probability that a healthy individual of age *x* survives as healthy up to *x+1*, the probability that a health individual of age *x* survives up to *x+1* (but becomes disabled during this period) and the probability that a disabled individual of age *x* survives as disabled up to *x+1*, by means of:

$$p_x^{HD} = w_x - q_x^{HD}, \quad (7)$$

$$p_x^{HH} = (1-w_x) - q_x^{HH} \quad (8)$$

$$p_x^{DD} = \frac{1}{t_x}\left[p_x - (1-t_x)\left[(1-w_x) - q_x^{HH}\right] - (1-t_x)(w_x - q_x^{HD})\right] \quad (9)$$