AGE-SPECIFIC CONTRIBUTIONS TO CHANGES IN THE PERIOD AND COHORT LIFE EXPECTANCY

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Abstract

The period life expectancy has lagged behind its cohort counterpart. To understand the disparity between the two measures we identify and compare the age-specific contributions to change in the two life expectancies. Using mortality models, we examine the effect of mortality changes over time. Finally, we apply our approach to historical data for Sweden. Preliminary results indicate a widening of the gap between the two life expectancies as a consequence of the dramatic mortality decline that occurred during the twentieth century. These first results also show that the divergence between the two measures is likely to become even greater in the future.

AGE-SPECIFIC CONTRIBUTIONS TO CHANGES IN THE PERIOD AND COHORT LIFE EXPECTANCY

Cohort life tables follow the mortality of a given birth cohort over its life course. Some industrialized countries have uninterrupted data spanning over several centuries allowing such analyzes. Period life tables, which require data from only one year, can depict the implications of more recent rates. Period life expectancies have been used for comparisons over time and across populations as a matter of accepted practice. However, Bongaarts and Feeney (2002) suggested that the conventional period life expectancy is inaccurate for countries with low mortality. The substantial gaps and lags between cohort and period life expectancies have been demonstrated by Goldstein and Wachter (2004). Here, we note that in nineteenth century Sweden the two life expectancies have similar values. Over time the disparity between them grows as increases in cohort life expectancies outstrip increases period life expectancies. Rather than reinforcing Bongaarts and Feeney conclusion that life expectancy is too large, we find that it underestimates current longevity.

In this paper we show that the reason for the inaccurate and low values of the period life expectancy is a change in the level and pattern of mortality over time. In the next section we bring definitions relevant to our study of mortality, followed with an examination of changes over time in life expectancies. Age-specific contributions to cohort and period life expectancies are calculated and compared using an age-decomposition of life expectancy. Mortality models are used to demonstrate how cohort and period life expectancies change under different mortality patterns. Finally,

applications to the mortality experience of nineteenth and twentieth century Sweden are provided.

COHORT AND PERIOD LIFE EXPECTANCY

The most commonly used summary measure of mortality is life expectancy. Period life expectancy at age *a* and time *t* is calculated as the number of life table person-years lived by the life table cohort above age *a* divided by the number in the life table cohort that survive from birth to age *a* under the rates of time *t*. For example, the period life expectancy at birth at time *t* can be expressed as

$$e_p(0,t) = \frac{\int_0^{\omega} \ell_p(a,t)da}{\ell_p(0,t)}, \qquad (1)$$

where $\ell_p(a,t)$ is the period life table survivorship function to age *a* under the rates at time *t*, and ω is the highest age attained. Letting the radix of the table be one, *i.e.* $\ell_p(0,t) = 1$, then $\ell_p(a,t)$ is the period life table probability of surviving from birth to age *a*. This probability is a function of the sum of the force of mortality at age *x* and time *t*, denoted as $\mu(a,t)$, between birth and age *a*

$$\ell_p(a,t) = \exp(-\int_0^a \mu(x,t)dx). \qquad (2)$$

The subindex *p* in equations (1) and (2) denotes that these are period measures. In the rest of the text *c* and *p* will be used to identify cohort and period respectively. For example, $\ell_c(a,t-a)$ is the life table probability of surviving from birth to age *a* for the cohort born at time *t-a* and $e_c(0,t-a)$ is that cohort's life expectancy. Here, it should be noted that at exact age *a* and time *t* the cohort and period force of mortality are equal

therefore $\mu(a,t)$ does not have a subindex. However, in contemporary low mortality countries there is a large difference between cohort and period life expectancies. Goldstein and Wachter (2004) show that the period life expectancy at year t is approximately equal to the cohort life expectancy for persons born half a century ago, or $e_p(0,t+50) \approx e_c(0,t)$.

Specific methods to analyze change in life expectancy over time have been developed by various demographers. A United Nations report (1982), Pollard (1982, 1988), Arriaga (1984), Pressat (1985) and Andreev (1982; Andreev et al. 2002) focused on discrete differences in life expectancy between two periods of time. Keyfitz (1977, 1985) considered as the time-derivative of life expectancy. Mitra (1978), Demetrius (1979), Goldman and Lord (1986), Vaupel (1986), Hokkert (1987), Hill (1993) and Vaupel and Canudas-Romo (2003) further developed this approach.

These decomposition methods differ in the election of the components of change, but they all consider age-specific contributions to the change in life expectancy. Thus, an interesting question is to examine how a change in mortality at a given age and time contributes to change in both period and cohort life expectancies. This leads to studying changes in life expectancies when the distribution and level of mortality are changing over time. To pursue these analyses, we follow some of the procedures developed in Vaupel and Canudas-Romo (2003).

PERIOD AND COHORT LIFE EXPECTANCY AGE-DECOMPOSITION

First we introduce two measures used in the developments that follow. Let the period probability density function describing the distribution of deaths (i.e., lifespans) in

the life table population at age a and time $t+a$ be denoted as $f_p(a,t+a) = \mu(a,t+a)\ell_p(a,t+a)$, and for the cohort born a years earlier $f_c(a,t) = \mu(a,t+a)\ell_c(a,t)$. Another measure needed in the derivations to follow is the rate of progress in reducing death rates, defined as the derivative of the logarithm of the force of mortality, $\rho(a,t+a) = \frac{\partial \ln \mu(a,t+a)}{\partial t}$. This age-specific rate is the same for the cohort and period because it is a function exclusively of the force of mortality. In the rest of the text, a dot over a variable is used to denote the derivative with respect to time of that variable, e.g. $\rho(a,t+a) = \frac{\dot{\mu}(a,t+a)}{\mu(a,t+a)}$.

Vaupel and Canudas-Romo (2003) show that an age-specific contribution to the change in life expectancy is equal to the product of three components. These components are the rate of mortality improvement at that age, the remaining life expectancy at that age, and the mortality density function at that age. For the period life expectancy, the age-specific contribution of age a, $\dot{e}_{p,a}(0,t+a)$, is thus

$$\dot{e}_{p,a}(0,t+a) = \rho(a,t+a)e_p(a,t+a)\,f_p(a,t+a). \qquad (3)$$

Adding those contributions over age gives the total change in period life expectancy, $\dot{e}_p(0,t+a)$. Paralleling equation (3), it is possible to define an age-specific contribution to the change in cohort life expectancy. The ratio of period to cohort age-specific contributions, $\dot{e}_{p,a}(0,t+a) \big/ \dot{e}_{c,a}(0,t)$, then allows us to see at which ages mortality changes affect cohort more than period life expectancy. Because the rate of mortality improvement $\rho(a,t+a)$, is the same in the period and cohort perspective, the ratio simplifies to

$$\frac{\dot{e}_{p,a}(0,t+a)}{\dot{e}_{c,a}(0,t)} = \frac{\rho(a,t+a)e_p(a,t+a)\,f_p(a,t+a)}{\rho(a,t+a)e_c(a,t)\,f_c(a,t)} = \frac{e_p(a,t+a)\,f_p(a,t+a)}{e_c(a,t)\,f_c(a,t)}.$$

Setting equation (1) for the remaining expectation of life at age *a*, and the probability density function describing the distribution of deaths of $f_c(a,t) = \ell_c(a,t)\mu(a,t+a)$ for the cohort and $f_p(a,t+a) = \ell_p(a,t+a)\mu(a,t+a)$ for the period we obtain

$$\frac{\dot{e}_{p,a}(0,t+a)}{\dot{e}_{c,a}(0,t)} = \frac{e_p(a,t+a)\ell_p(a,t+a)\mu(a,t+a)}{e_c(a,t)\ell_c(a,t)\mu(a,t+a)} = \frac{\int_a^\omega \ell_p(x,t+a)dx}{\int_a^\omega \ell_c(x,t)dx}. \qquad (4)$$

The ratio of period over cohort age- specific contributions simplifies to the ratio of person-years lived above age *a.* Using the life table notation for this measure we have $T_p(a,t+a)/T_c(a,t)$. In other words, the ratio of total number of years remaining to the synthetic cohort (period) until the last member dies, over those for the cohort. This is a surprising result because the comparison transforms to a fraction that does not contain our explicit measure of change, $\rho(a,t+a)$. In equation (3) the age-specific contributions of the change in period or cohort life expectancy includes that rate of mortality improvement, but it drops out of the final relationship in equation (4).

To gain an appreciation of how the period and cohort age-contributions differ, the following section presents a continuous model where mortality changes over age and time at constant rates.

PERIOD AND COHORT MODELS OF MORTALITY

Model populations provide a useful way to examine the age-specific contributions to changes in period and cohort life expectancy. The formulation used here is an

extension of the Gompertz model of mortality where there is an infant mortality component and a continuous rate of decline over time. This model is a combination of the model proposed by Siler (1979) and the continuous rate of decline model discussed by Vaupel (1986) and Schoen et al (2004). The force of mortality at age *a* and time *t* is defined as

$$\mu(a,t) = \exp[a_1 - b_1 x - c_1 t] + \exp[a_2 - c_2 t] + \exp[a_3 + b_3 x - c_2 t], \quad (5)$$

where there are three constant terms which reflect the value of $\mu(0,0) = \exp[a_1] + \exp[a_2] + \exp[a_3]$; parameters b_1 and b_3 are fixed rates of mortality decline and increase over age, respectively, which account for infant and senescent mortality; and parameters c_1 and c_2 are the constant rates of mortality decrease over time.

In the model we begin with fairly high infant mortality using values of $a_1 = 0.2, a_2 = 0.003$ and $a_3 = 0.000015$. The early decline over age proceeds at a pace of $b_1 = 1$ with an overall increase with age at a pace of $b_3 = 0.1$. These values have been adapted from a comparison of the Siler model with the different model life tables elaborated by Coale and Demeny (Gage and Dyke, 1986). The pace of mortality improvement is almost one percent at older ages but historically it was more accelerated at younger ages. Here we have chosen $c_1 = 0.02$ and $c_2 = 0.01$.

Figures 1ab show the Lexis surfaces of the age-specific contributions for the change in cohort and period life expectancy respectively. (Lexis surfaces have been described by Andreev, 1999.)

[FIGURE 1ab about here]

The cohort and period patterns in Figure 1ab are very similar. Initially changes at early ages have a great impact on overall life expectancy, but that diminishes over time. Apart from the highest ages, the age-specific contributions reach a minimum between ages 20 and 50 and then increases to a maximum between ages 70 and 90. Finally, at the very high ages there is a pronounced decline in impact.

Figure 2 shows the Lexis surface for the ratio of (i) the age-specific contributions to the change in period life expectancy over (ii) the corresponding age-specific contributions to the change in cohort life expectancy.

[FIGURE 2 about here]

As shown in equation (4), at each given age this ratio is equal to the ratio of period to cohort person-years lived above that age.

Figure 2 shows that this ratio increases with age. For younger ages it is below one, indicating that changes at those ages have more impact on cohort than period life expectancy. Young cohorts at time *t* will experience death rates at older ages lower than those experienced by older persons at time *t*. Therefore, changes in mortality for these young cohorts are reinforced by later mortality declines not present at time *t*. The inverse process occurs at older ages where the values of the ratio are above one. The cohorts that reach advanced ages at time *t* have experienced death rates at earlier ages greater than those seen in the period *t.* As a result, past changes in mortality have greater impact on life expectancy than future improvements.

Figure 2 shows the steady increase over time in the age at which the ratio is 1. In the first years, most of the values of the ratio are above 1. At time 140, most of the ratios are below 1. The greatest changes occur at younger ages where the ratio declines

markedly over time. The smaller number of deaths at young ages under low mortality regimes reduces the potential contributions from young ages, especially with regard to period life expectancy.

Figure 3 shows the total annual amount of change in the cohort and period life expectancies and the accumulation of these changes over time starting at time zero.

[FIGURE 3 about here]

Cohort annual change and cohort accumulated change are greater than the period values. This occurs even in year 140 when infant mortality is low and life expectancy is near 110 years.

EXAMINING COHORT AND PERIOD MORTALITY IN SWEDEN

To assess age-specific contributions to period and cohort life expectancy in Sweden, we use data derived from the Human Mortality Database (2004). To calculate the age-specific contributions to cohort life expectancy for cohorts that are not extinct we have extrapolated future mortality. That was done using a continually declining mortality model, similar to that in equation (5), with a constant pace of decline of c=0.005. For example, if the last year with available data is 2000, then the age-specific death rates for that year, $\mu(a,2000)$ are the base of the model. For age a and year $t > 2000$ the force of mortality is

$$\mu(a,t) = \mu(a,2000)e^{-c(t-2000)}. \qquad (6)$$

Figure 4 presents the cohort and period life expectancies in Sweden 1751-2002 with extrapolated cohort values for the years 1912 to 2002.

[FIGURE 4 about here]

During the last years of the century the fluctuations observed in the period measure have largely disappeared. The estimated values of the cohort life expectancy, with the selected modest decline over time, correspond to a gap of almost 10 years between the two life expectancies. Before 1900, however, that gap was much smaller. For Sweden between 1900 and 2002, Figure 5 shows the Lexis surface for the ratio of the age-specific contributions to period relative to cohort life expectancies, as calculated from equation (4).

[FIGURE 5 about here]

Similar results as those for the Siler model in Figure 2 are found now in Figure 5 for Sweden. The young ages are dominated by higher values of cohort age-specific contributions while at older ages the period measure finds higher values. There is also a clear shift of the crossover value of 1 towards older ages, from age 20 at the beginning of the twentieth century to age 60 at the end of the century.

The gap observed between cohort and period life expectancies begins at the end of the nineteenth and first decades of the twentieth centuries in Sweden. This is also the period of great improvements in infant mortality which advantage the increase in the cohort measure. At the other end of the twentieth century, improvements in mortality at older ages have gained in importance, but the gap between the two measures keeps enlarging.

The change in age-specific contributions to period and cohort life expectancy reinforces the importance of mortality trends at the older ages. The extent of future mortality improvement will determine how the gap between period and cohort life expectancy will change.

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Figure 1a. Age-contributions to the change in the cohort life expectancy in the Siler model.
Two rates of decline over time C1=0.02 and C2=0.01

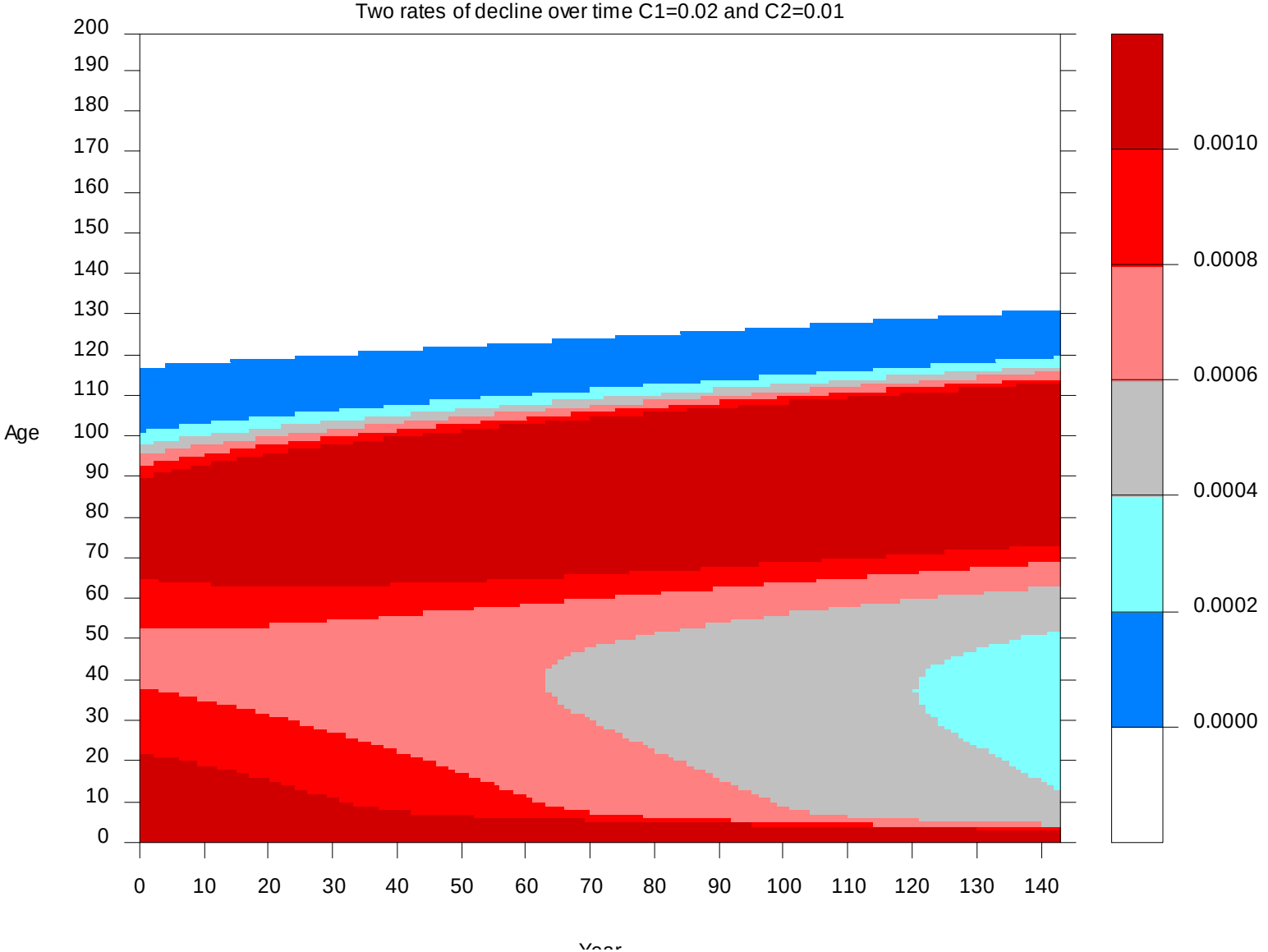

Figure 1b. Age-contributions to the change in the period life expectancy in the Siler model.
Two rates of decline over time C1=0.02 and C2=0.01

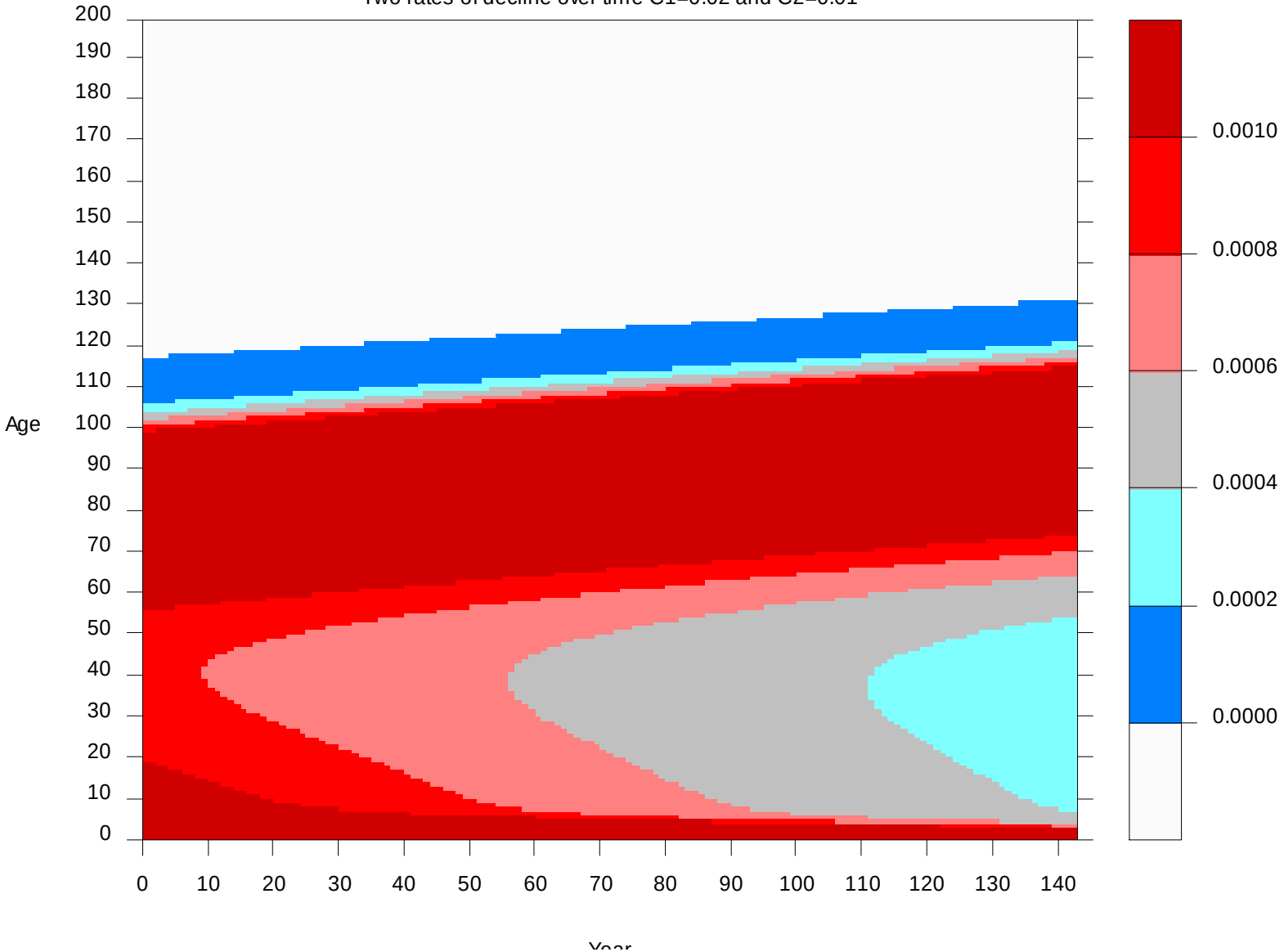

Figure 2. Ratio of age-specific contributions to the change in the period life expectancy over the change in the cohort life expectancy, Siler model with rates of decline over time C1=0.02 and C2=0.01

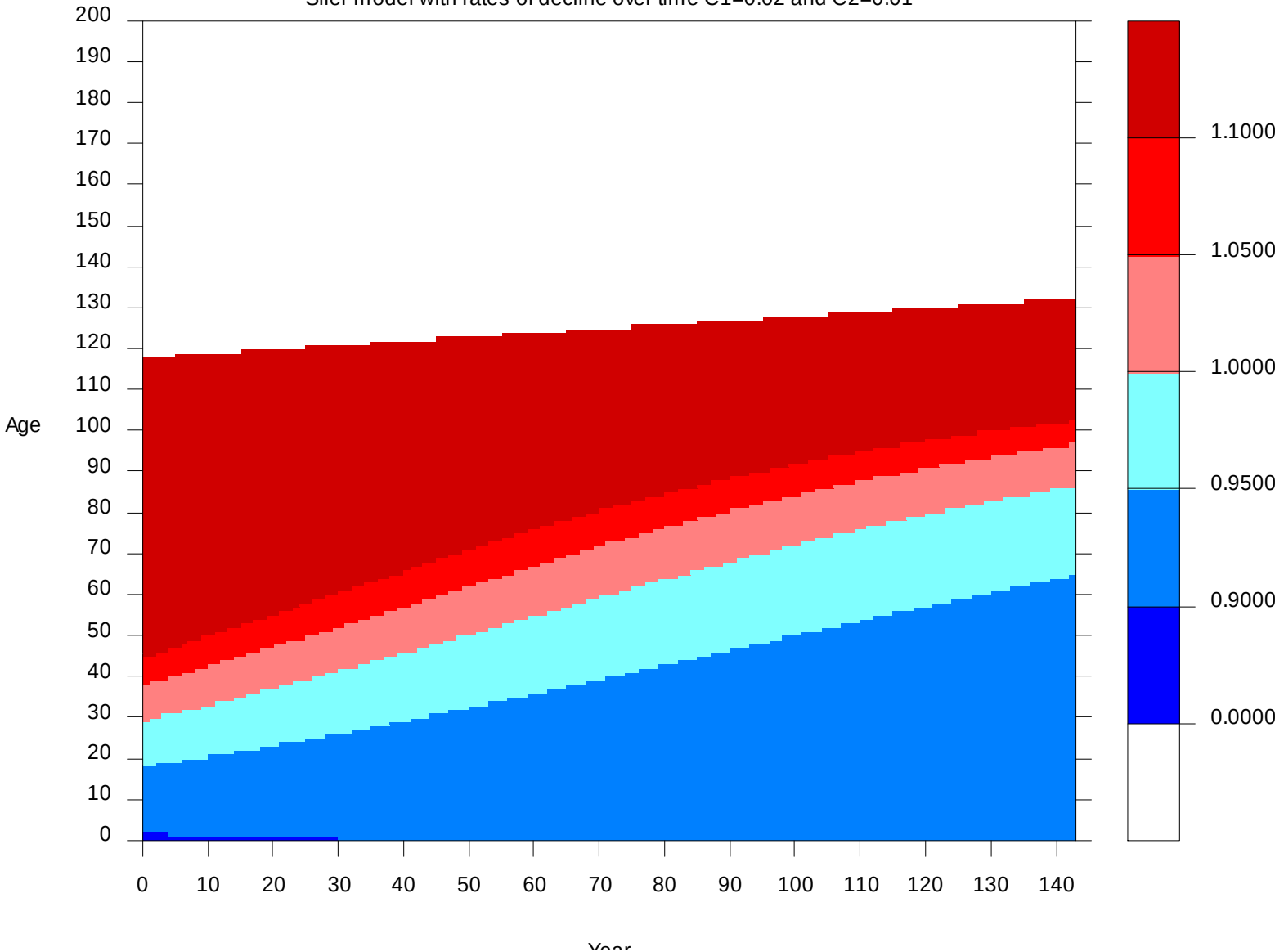

Figure 3. Cohort and period life expectancy annual and accumulated changes in a Siler model with rates of decline over time C1=0.01 and C2=0.01.

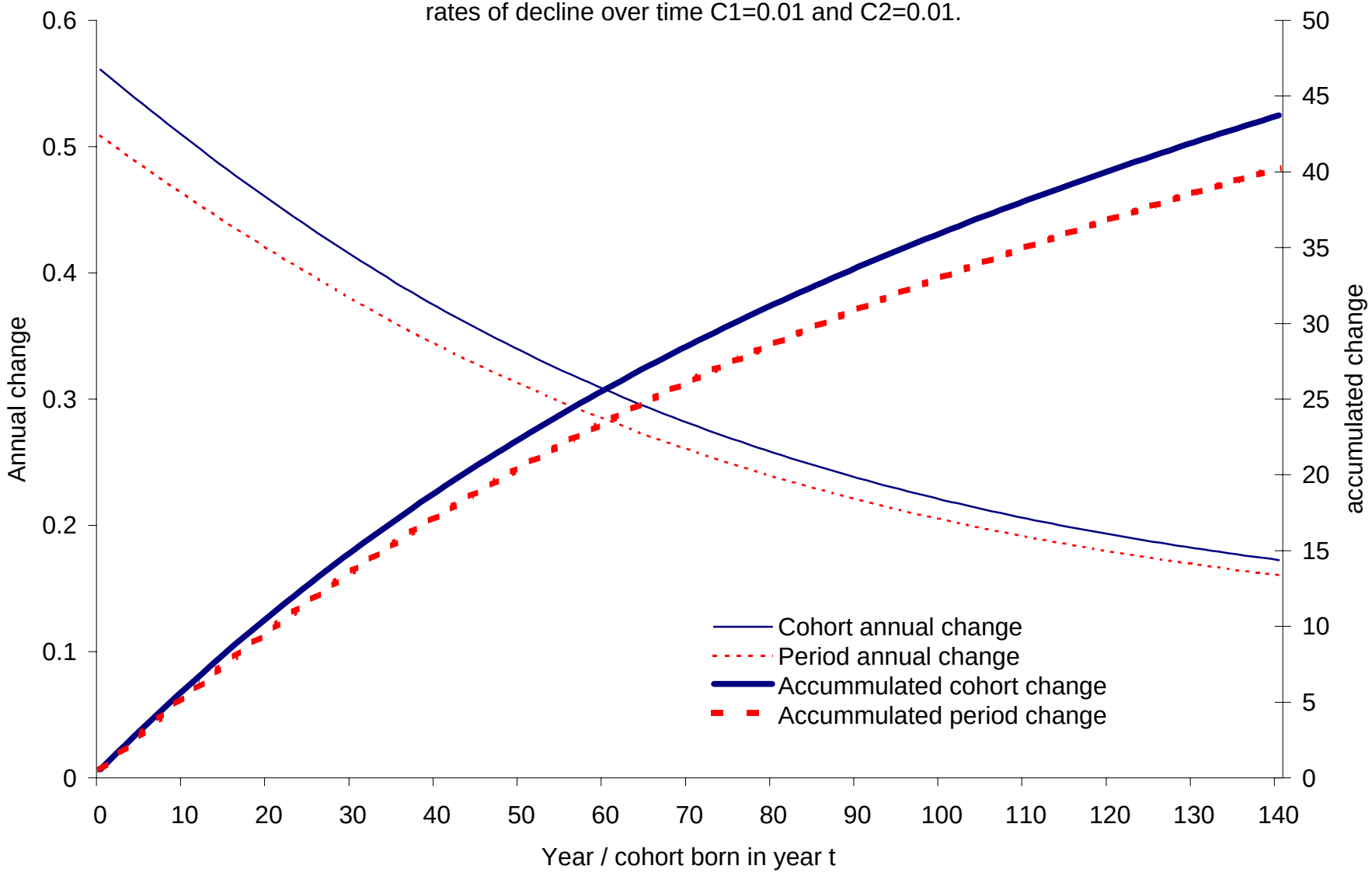

Figure 4. Cohort and period life expectancy, with cohorts completed with a continuous declining pace in mortality of C=0.01, Sweden.

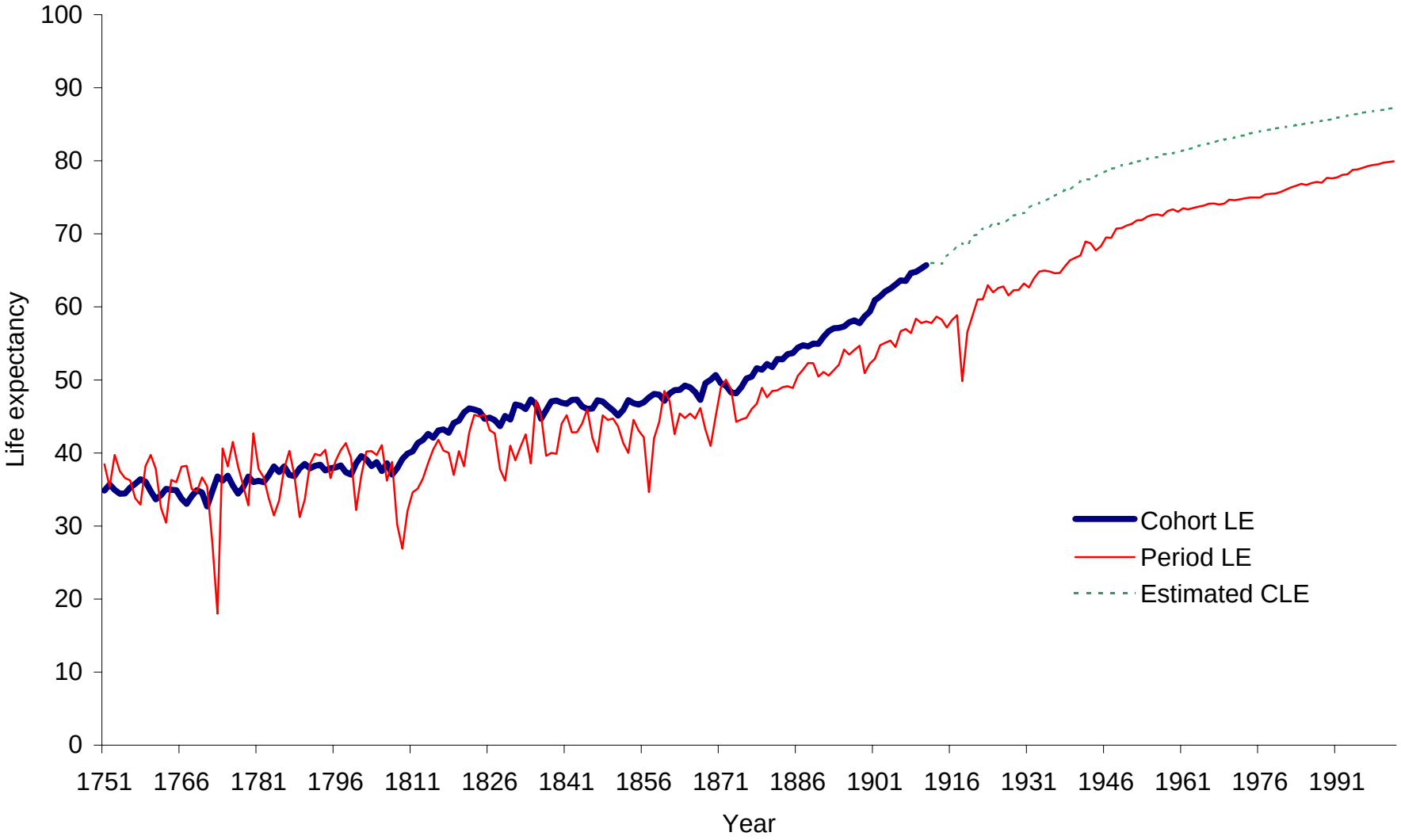

Figure 5. Ratio of the period over the cohort age-contribution, Sweden 1900-2002.
Cohorts completed with a pace of decline over time of C=0.01