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How to get a robust estimation of the modal length of life?

Siu Lan Karen CHEUNG*, Jean-Marie ROBINE** and Graziella CASELLI***
*DEMOG, RTN post-doc
**INSERM, Health and Demography
***University of Rome “La Sapienza”

EXTENDED ABSTRACT

The natural or normal length of life corresponding to ageing deaths and its links to potential limits of human lifetime has been a question of great interest among biologists, statisticians and demographers. Historically, the determination of aged persons died purely of “*old age*” was uncertain because casualty and ageing are hardly disentangled (Graunt, 1662). Human being surviving to one hundred years is the rare exception (Buffon, 1714). A theoretical limit of human life span was placed at various ages such as 86, 90, 95 or 100 restricted by a last observation in the life table (De Moivre, 1756). Questions related to the natural and ordinary duration of human life and its extreme limit have been a major concern in the past (Flourens, 1855; Thoms, 1873).

Lexis (1878) synthesized Quetelet’s model of “*l’homme moyen*” and the law of normal distribution of errors to study human mortality that the length of life distribution (*d(x)* in a life table) are divided into three parts: (1) a J-curve right after birth corresponding to infant deaths; (2) the normal deaths around the late modal length of life which obey the law of accidental errors reflecting the natural lifetime; and (3) the deaths occurred in a transitional region overlapping of premature deaths and normal deaths.

By using the symmetrical property of the hypothetical normal distribution of ages at death, the first half distribution above the mode allow to distinguish the normal deaths from premature adult deaths to form the second half of the normal distribution. The normal life duration should be stable placing between 70 and 80 years. For Lexis the normal length of life is not equated with the mean length of life. The latter, a purely arithmetical term, and in no way represents a true mean characteristics of the mortality of the human species which produces bias in the estimate of the length of life because infant and young mortality bring life to a premature end. But since the 1920s, the life expectancy at birth became more prevailed over the modal length of life to a measure of human longevity. It was considered as the best index of the life span (Dublin, 1923).

As human life expectancy at birth is now approaching 85 years in the low mortality countries, already reaching 85.2 years for Japanese women in 2002 (Statistic Bureau, 2004),

using life expectancy at birth to examine whether there is an upper limit to the human life span becomes dubious. Recent and future gains in human life durations are mainly due to the reduction of mortality among the old and oldest-old people (Kannisto et al., 1994; Horiuchi, 2000; Tuljapurkar et al., 2000; Wilmoth, 1998, 2000, 2002; Robine 2001). Life expectancy at birth does not give a sufficient account of recent trends in old age mortality decline (Manton et al., 1999; Kannisto, 2001; Wilmoth, 2000, 2002; Hayflick, 2002; Veron and Rohrbasser, 2003) because infant and premature deaths are included in life expectancy at birth. An arbitrary selection of age limits in term of age-specific death rates, life expectancy at birth or probabilities of dying and of survival is usually required to specific to ageing mortality, but not satisfactorily. Longevity measured by the maximum life span is strongly affected by the sample size.

On the contrary, the modal length of life has been remarkably appreciated in the past, but not commonly practiced today. The re-emergence of the concept of normal life durations and the use of the late modal length of life and the standard deviation above it for summarizing the normal longevity are due to the last work of Kannisto (2001). Cheung et al. (200X) pursued his work and introduced $M \pm 4SD(M+)$ that represent the shortest and longest normal life durations, provide useful indicators for studying human longevity under a given mortality regime, unaffected by premature deaths and free from bias caused by any arbitrary age limits.

In this paper, therefore, we verify the relationship between the increase in the age of late modal length of life (M) and the decrease in the standard deviation above it (SD(M+)) in France, Italy, Japan and Sweden. Although the increase in M and the decrease in SD(M+) moved linear for the last 25 years in Hong Kong, this linear reduction in SD(M+) would not expect to last in the long run because the value of SD(M+) could not reach zero. Human survival curves will never become totally rectangular (Wilmoth and Horiuchi, 1999). Hence, we propose to use a log-linear model instead of a linear model to fit and predict the value of SD(M+) when M is increasing. Our hypothesis is that SD(M+) will be changed proportionally to the increase in M. M+4SD(M+) will be increasing accordingly which implies a minimum human longevity extension associated with the ultimate increase in M.

More importantly, due to a certain flatness and large confidence interval in the distribution of ages at death (d(x) curve) near its highest point in life tables, the mode is sensitive to minor variations in the curve which makes it important that the curve is unimodal and relatively smooth. A quadratic fitting is thus used to smooth only for +/- five year ages in the region of the late mode to obtain a robust estimation of the mode in the hypothetical normal distribution (fitted mode) and its immediate surrounding values (fitted dx +/- five years from the observed mode).