Patterns of Mortality Improvement over Age and Time in Developed Countries: Estimation, Presentation and Implications for Mortality Forecasting

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Abstract

In this work we estimated surfaces of mortality improvement over age and time for 18 developed countries, and for the period from 1950 to the latest year available. The estimation has been

carried out by the tensor product spline tested in advance on simulated data. The results suggest that mortality did not decline uniformly across the countries rather the patterns of mortality improvement can be classified into two broad categories: predominately period and age shifting pattern of mortality decline. In addition, strong cohort-like patterns in mortality improvement rates, both positive and negative, have been detected for several populations. These findings have a potential importance for building better models of mortality forecasts. Usefulness of presented techniques for mortality forecasting field is discussed and illustrated on several examples.

Introduction

Many objectives of population research require investigation of temporal dynamics of death rates. Changes in mortality over time are usually analyzed by calculating rates of mortality improvement. Typically such rates are computed over extended periods of time, five or ten years, and over broad age groups, usually five year age groups. Even if the underlying population data are available by single calendar year and by single year of age the shorter periods and age groups are seldom used because the resulting estimates are often unstable. By adopting this approach for investigating mortality decline over time the finer details of mortality dynamics are necessary lost potentially preventing deeper understanding of the observed mortality trends and consequently hindering development of adequate models for exploration, comparison and forecasting of death rates.

In this article we present an alternative approach for exploration temporal dynamics of death rates over age and time based on estimating a *surface* of mortality improvement. We discuss the method in details and show that exploration of such surfaces can be highly facilitated by techniques for visualization demographic data (Vaupel; Wang; Andreev, and Yashin 1998). After testing the performance of the proposed method on simulated data we apply it for exploration patterns of mortality improvement in the developed countries (Human Mortality Database 2004). Subsequently, we provide a brief discussion and classification of the observed patterns of mortality improvements. Usefulness of the proposed techniques for field of mortality forecasting, and generally for analysis of mortality surfaces is discussed as well.

Method

Let D_{xt}, E_{xt}, $m_{xt} = \frac{D_{xt}}{E_{xt}}$ to be deaths, exposure estimates and death rate at age *x* and in the year *t*, respectively. We define an estimate of rate of mortality improvement at age *x* and year *t* as

$$r_{xt} = -\ln \frac{m_{x,t+1}}{m_{x,t}} \tag{1}$$

Direct examination of the matrix of r_{xt}'s obtained by applying (1) is not possible due to high variability of the estimates. Fig. 1 shows Lexis map (Vaupel; Wang; Andreev, and Yashin 1998) of such matrix for female population of Japan (see also data section and additional interpretation of Lexis maps below in text). Even if some patterns are discernable high variation of estimates prevents clear understanding of the raw surface of mortality improvement as estimated by (1). We need therefore to smooth this surface in order to reveal underlying pattern of mortality improvement over age and time. By "smoothing" we imply a usual statistical paradigm that the observed surface of mortality improvement shown in Fig. 1 is a distorted image of a "true" surface of mortality improvement, generally smooth and slowly changing function of age and time. The distortion itself is caused by some random noise process with generally unknown properties. There are many ways to proceed from this point but in essence we are trying to satisfy two mutually exclusive conditions: to provide the best estimate of the "true" surface of mortality improvement without making any strong assumptions about the underlying function.

A promising candidate method to proceed with is so-called tensor-product spline (de Boor 2001). The idea is to smooth the r_{xt} surface with this method in a hope that it would decompose the r_{xt} surface into the "true" surface of mortality improvement and the noise component. The "true" surface of mortality improvement is then expected to be well approximated by the tensor-product spline.

Here, we provide a brief overview of this surface smoothing technique, more information can be found in (de Boor 2001) or in (Dierckx 1993). The method can be considered as an extension of Whittaker-Henderson graduation (Whittaker 1923) to two dimensions, well known and frequently used in actuarial science (London 1985).

For univariate data the smoothing method can be formulated as follows. Given observed values of rates of improvement r_t find a function $\hat{r}_t$ that for a given parameter *p* (not unknown in advance) minimizes

$$p\sum_t (r_t - \hat{r}_t)^2 + (1-p)\int_{t_0}^{t_n} (\hat{r}_t^{(2)})^2 \, dt \qquad (2)$$

over all functions with continuous second derivatives. The r_t is sequence of observed rates of mortality improvement over period $[t_0, t_n]$ e.g. computed by (1) for a single age, and $\hat{r}_t^{(2)}$ is the second derivative. The first term in (2) measure deviation between observed r_t and smoothed values of $\hat{r}_t$, and the second term is provides a measure of roughness of $\hat{r}_t$. For a given value of smoothing parameter *p* we need to find a function $\hat{r}_t$ which provides a best approximation (in terms of squared differences) to the sequence r_t simultaneously minimizing the roughness criterion given by the second term.

Usually such conditions are mutually exclusive: the better approximation requires rapidly varying functions but the roughness term will be necessarily increased in this case. This is the case, for example, is the error term is given by the *white noise sequence*. The normalized spectrum of the white noise sequence is a constant value equal to 1 for all frequencies (Diggle 2000). The closer approximation would require closer approximation of high frequencies increasing thereby the roughness term.

Surprisingly enough, the solution of this variational problem turned out to be a natural cubic spline with simple knots at t_i (de Boor 2001). The idea of such function approximation, by balancing the approximation and roughness criterion, can be traced back to (Whittaker 1923). He considered uniform knot sequences and used divided differences instead of derivatives. The formulation (2) is taken from (de Boor 2001); we did omit weights and used cubic splines only. Assumption of homoscedasticity might be relaxed later but we are not dealing with it here.

Idea of extension of smoothing spline to the higher dimensions stems from representation of any spline of order *k* (k=4 for cubic splines) as linear combination of B-spline functions (basis spline functions) of the same order *k* and for the knot sequence *t*, $B_{i,k,t}$. In this case the smoothed sequence of univariate rates of mortality improvement $\hat{r}_t$ being a cubic spline as a solution of (2) can be represented as

$$\hat{r}_t = \sum_i c_i B_{i,t} \tag{3}$$

The B-spline functions for given knot sequence *t* are fixed so smoothing spline (2) is determined entirely by sequence of B-spline coefficients c_i.

Eq. (2) can generalized to two dimensions by forming a tensor product as

$$\hat{r}_{tx} = \sum_i \sum_j c_{ij} B_{j,x} B_{i,t} \tag{4}$$

Similarly to the univariate case the smoothed estimates of rates of improvement $\hat{r}_{tx}$ over age and time are determined entirely by *matrix* of B-spline coefficients c_{ij}.

Representing of $\hat{r}_{tx}$ in form of (4) implies that the surface $\hat{r}_{tx}$ is given by a cubic spline both in time and age direction, and for any Lexis square defined by *[t, t+1]* and *[x, x+1]* the surface is represented by cubic polynomials in any direction. Such polynomial patches are "glued" together by imposing usual spline smoothness conditions so the partial derivatives are continuous up to the second order.

The matrix c_{ij} can be found by employing algorithms for solving univariate problem (2) significantly simplifying computations. To compute the c_{ij} matrix we can start, for example, with smoothing in time direction by applying (2) to series of mortality improvement rates at each age. This operation will produce a surface which is smoothed in the time direction but still rough in the age direction. By smoothing the resulting matrix of *B-spline coefficients* in the age direction we will obtain estimates of the c_{ij} matrix, and consequently, form (4) of the smoothed surface $\hat{r}_{tx}$.

Before applying the tensor-product spline for estimating $\hat{r}_{tx}$ we need to select the smoothing parameter *p*. This parameter can be different for age and time directions but we use the same value for both directions implying that we need to select one value of *p* before smoothing. Choice of this smoothing parameter is not straightforward as it should reflect out belief about smoothness of the underlying surface of mortality improvement. Generally, without making additional assumptions about the data, we can select any value of this parameter in the range between 0 and 1. The values of *p* close to 1 will make the spline to approximate the observed data more closely; for p = 1 the spline would interpolate the data so we would fit data exactly. For *p* approaching to zero the roughness term in (2) will be gaining more importance

making the spline to be smoother. In univariate case the smoothing spline converges to the least square line (we speak of convergence as the first term in (2) does not depend on observed data if p = 0). In the two dimensional case the spline converges to bilinear polynomial $\hat{r}_{xt} = c_0 + c_1 x + c_2 t + c_3 xt$ which is the tensor product of two linear functions. In each direction therefore the $\hat{r}_{xt}$ will be represented by a linear function and for $x=t$ by a quadratic function if $c_3 \neq 0$ (de Boor, personal communication).

Automatic selection of smoothing parameter in univariate case has drawn a lot of attention in statistics over last years . Several methods e.g. generalized cross-validation have been proposed for automatic choice of the smoothing parameter *p*. The research focused mostly on univariate case e.g. for problems of density estimation. It is not clear how such method should be applied in our problem setting. It is also known that in many practical situations such methods have a tendency to undersmooth the data . We therefore conducted a simulation study in attempt to portray a real world situation which should help us with choosing the value of smoothing parameter. The complete description of the study is still pending, only main results are presented here.

In the simulation study we assumed that rates of mortality improvement might follow one of pre-defined patterns, and after adding random noise we tried to recover the pattern with different values of the smoothing parameter. One of the patterns we dealt with implies that the rates of mortality improvement are constant over time but can change over age according to some age-specific profile. Figure 2a) shows the selected age specific profile: it has maximum at age 0, with rate of improvement about 3.2%, it also has a maximum at age about 42 with the rate about 1.7% and maximum at 80 with 1.3%. This profile was estimated by fitting Poisson regression to female population of United States, over period 1959-1999, and subsequent slight smoothing over age. Figure 2b) show pattern of mortality improvement over age and time used in this simulation. We used the Lexis map display (Vaupel; Wang; Andreev, and Yashin 1998) so it is easy to grasp the pattern represented in this figure. Recall, that by design the surface is not changing over time so we can see only horizontal stripes corresponding to the different levels of mortality improvement over age, Figure 2a). The deepest magenta color, for example, corresponds to the scale level 0.02 and over, and such rates are found only at ages below 10. Selection of scale levels is an arbitrary process, and usually they selected to highlight different aspects of the data. If the main concern is exploration of the underlying pattern the scale levels

can be selected equal to the percentiles of the underlying matrix of rates of improvement. In this case all colored areas of the Lexis map will be approximately equal significantly facilitating perception of the surface. We used exactly this approach to select the scale levels in Fig. 2.

In process of evaluating of tensor product spline procedure for recovering underlying pattern of mortality improvement we a) selected some initial age schedule of mortality b) applied the simulated surface of mortality improvement to reconstruct surface of death rates c) added uncorrelated random noise ($e \sim N(0,0.04)$) to every death rate, and c) applied (1) to compute observed or raw surface of mortality improvement rates r_{xt}. Because of the added noise death rates it is not anymore as smooth as that shown in Fig. 2 b) rather it appears closer to that shown in Fig. 1 with a hardly discernable pattern. Our next step was to compute smoothing tensor product spline with different value of the smoothing parameter. For each spline we computed integrated square error ($ISE = \sum (r_{xt} - r_{xt})^2$) which measures the distance between raw r_{xt} and smoothed $\hat{r}_{xt}$ surfaces: the close is approximation the lower is *ISE*. Fig. 2c) shows dependence of the ISE on the smoothing parameter (we used *–logit(p)* as abscissa in Fig. 2c) so the degree of smoothing increases from left to right).

One can see that at first the ISE declines—this correspond to the case when the smoothing procedure indeed eliminates some noise so the smoothed matrix $\hat{r}_{xt}$ approximate the simulated matrix of mortality improvement rates more closely. The best approximation is reached at minimum of the ISE curve (ISE = 0.005, p=0.002 and –logit(p)=6.18). For higher degree of smoothing the ISE increases. This corresponds to so-called oversmoothing when the fitted spline is smoother than our simulated surface of mortality improvement. The ISE tends to an asymptote implying that the tensor product spline converges to bilinear polynomial for high degree of smoothing (see also above) so further changes in the smoothing parameter do not change the smoothing spline in any appreciable way. Finally, Fig. 2d) shows the estimated surface of mortality improvement with p = 0.002 at minimum of the ISE. This is the closet approximation to the underlying surface of mortality improvement we can get in this simulation. By comparing Fig. 2c) and Fig. 2d) we can see that with some distortions we can recover the underlying surface of mortality improvement. This approximation could be better if we would smooth more over time and less over age but it would involve additional assumptions about the underlying surface of mortality improvement—something we are trying to avoid in this work.

As it follows from this and other (not reported) simulations the smoothing parameter is expected to be of order approximately 10^{-3}. Lower values of this parameter will tend to oversmooth the data while higher levels to undersmooth. Perhaps undersmoothing is less critical as we might fail to recognize some patterns possibly hindering advancing our knowledge while oversmoothing might produce some spurious patterns leading potentially to wrong conclusions. Therefore for each population we decided to produce a hundred smoothed maps of mortality improvement patterns to evaluate visually in a movie-like fashion how pattern of mortality improvement changes with value of the smoothing parameter p. As it turned out that the value of $p = 0.02$ appears to provide an adequate degree of smoothing and bearing in mind that we likely to undersmooth the data we decided to use this value of smoothing parameter in applications to real data.

In sum, the method we used for smoothing is an extension of the smoothing spline (Whittaker 1923;Schoenberg 1964;de Boor 2001) to several dimensions. It is intended for interpolation and smoothing of the girded data making it is well suited for the data included in Human Mortality Database. To apply this method one needs to select order of splines in year and age direction and smoothing parameters in each dimension. We used cubic splines and a single smoothing parameter equal to 0.02.

Data

All data except the data for Australia are from Human Mortality Database (Human Mortality Database 2004), the Australian data are from (Australian Demographic Databank). Data for the following populations have been included in the analysis: Australia, Austria, Canada, Denmark, England & Wales, Finland, France, Germany (East), Germany (West), Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, United States.

Human Mortality Database includes death counts, exposure at risk and death rates for more than 20 developed countries. The data are available by single calendar year and by single year of age. For this analysis we utilized only data starting with year 1950 and up to the latest year available. The death rates above age 100 have been aggregated in an open age group 100+. All population data used here were subject to rigorous data quality checks, and the quality was found acceptable for the proposed analysis.

It is important to note that the data used here are themselves estimates based on raw population and death data which are specific for each country. For recent years usually no adjustments required to obtain death counts and exposure estimates by single year of age and by single calendar year but for the earlier years such adjustments might be considerable. The original raw data is also publicly available in Human Mortality Database, and the methods for obtaining mortality estimates are described in (Wilmoth 2002).

Australian data is very similar to the population data included in the Human Mortality Database. The complete description of this database is pending.

Patterns of Mortality Improvement over Age and Time

In this section we present patterns of mortality improvement in developed countries over age and time. We also suggest an approximate classification capturing the gist of the observed patterns. Surfaces of mortality improvement have been estimated by tensor product smoothing spline with the smoothing parameter equal to 0.02. Lexis maps of the estimates are included in Fig. 3. This figure includes a great deal of material on mortality improvement presented in a concise and revealing form. An interested reader is invited to examine them for him/herself as the only features of mortality dynamics relevant for this project are discussed here. Our main goal is to provide a reader with clear and informative presentation of mortality dynamics over age and time simultaneously highlighting general patterns.

We start with interpretation of Lexis maps presented in Fig. 3. All maps share the same scale provided at the bottom which divides all rates of improvement according to percentage if increase or decline in death rates. Negative rates of improvement (years and ages where death rates were increasing) are depicted in blue and positive rates of improvement in magenta. This major color breakdown permits to see a glance where death rates were declining and where they were on a rise. Deepness of each color is associated with a magnitude of decline or increase in mortality. Let us look at the Lexis map for United States, Females (Fig. 3). From the scale legend we can see that magenta hues  etc. are assigned to the following ranges of values: 0–0.01, 0.01–0.0198, 0.0198–0.0296 etc. Therefore all rates of improvement between 0 and 0.01—above zero but less than 1%—are painted in the first magenta hue  . The second, more deep, magenta hue  , is for rates of improvement between 1% and 2%. Generally, the deeper is color the stronger are rates of improvement. If we look at the Lexis map—in the early

1950s female mortality was declining virtually at all ages with the highest rates of decline (>5%) are observed at ages 10–35. Similar to the magenta part of the scale, the lightest blue color  expose *increases* in death rates on an order of magnitude between 0 and 1%; the next blue hue  corresponds to 1%-2% increase in death rates and so on. In the late 1990s, for example, death rates were increasing among U.S. females in their forties (<1%), and among those aged 80 and over. In a similar way one can interpret the Lexis maps for all other populations shown in Fig. 3.

The first pattern, which is called hereafter a *predominantly period pattern of mortality decline*, is characterized by more or less stable age-specific pattern of mortality improvement prevailing over extended period of time, 10 or more years. The age pattern of decline might be changing from period to period and it might vary among populations but important feature is that the periods with relatively stable rates of improvement are easily detectable. It can be hypothesized that mortality progress in this case is governed mostly by period factors.

Such pattern is observed, for example, in United States. In the beginning of the 1950s death rates were declining virtually at all ages with higher rates of improvement observed at younger ages (increase in death rates at ages 95 and over as indicated by a blue spot in the upper left corner is likely due to improvements in data quality). Over time the process largely decelerated and in the 1960s mortality started to increase. Male death rates were increasing nearly at all adult ages with the highest rates of increase observed in the earlier twenties. Among females death rates were on a rise only at ages 15–50 while at other ages mortality continued to decline. Another important change of age-specific pattern of mortality decline occurred in the 1970s. A rapid decline took place both among males and females at a rate 1.5-2% per annum or higher. The rates of improvement were the highest at ages below 10, at age 38 and, for females, at age 80. In the 1980s the progress against mortality slowed down again. Mortality did continue to decline at a noticeable pace at ages above 50 and below age 10 but the magnitude of decline was not as strong as in the 1970s. Over this period we also observe increase in death rates at ages 30–40: stronger for males and less that for females. Adverse trends in male mortality reversed in the 1990s when male death rates resumed to decline. Similar to the 1970s the highest rates of improvement are observed in three separate age groups: at ages 5–15, 30–40 and 60–70. Dissimilar to males the female pattern of decline did not undergo substantial changes in the 1990s except for some acceleration of decline at ages below 20 and increase in mortality at

very old ages, above 80. As a decade ago at ages above 60 the rates of improvement were moderate, generally less than 1%. We also observe increase in mortality at ages 40–50, somewhat along cohort lines centered on the cohorts born in the mid-1950s. No such cohort-like effect is found for males.

The predominantly period pattern of mortality change is also clearly observable in the Canadian data. Somewhat surprisingly Canadian pattern of mortality improvement appeared to be very similar to the U.S. pattern (Fig. 3). It does not mean that rates of improvement were the same in both populations—Canadian rates of improvement are consistently higher than that in United States—rather age-specific patterns and timings of changes match closely. Mortality developments in Canada were more favorable than in United States: if death rates were declining they were declining faster in Canada and if increasing the rate of increase was lower than in the United States. Unfavorable developments in the U.S. female mortality in the 1980s and 1990s are not found in Canada but we do observe some increases in death rates at the highest ages together with attenuation of mortality decline in the cohorts born in the mid-1950s. Even if the level of mortality in Canada is lower than in the United States the mechanisms governing temporal mortality dynamics seems to be very similar in two countries, likely due to geographical proximity.

Among European countries this pattern of mortality improvement is found in Finland, Norway, Spain, in the female populations of the Netherlands and Sweden, and in former East Germany. There is a high variation in age-specific schedules of mortality improvement in different populations and periods but the estimated surfaces of mortality decline (Fig. 3) seem to fit the predominantly period pattern very well. In Norway and Finland, in the beginning of the 1950s, male mortality was declining at all ages. Noticeably higher rates are observed in Finland and at younger ages. Shortly after mortality conditions deteriorated, the death rates at ages above 40 in both countries started to increase and mortality progress at lower ages slowed down. In the 1970s decline in death rates resumed again with significantly higher rates of improvement in Finland than in Norway. Among children and infants (ages≤10) mortality rates were declining very fast in both countries: on average 7.4% in Finland and 5.2% in Norway. At adult ages (>40) Finnish declines were almost double that of Norway: on average Finnish death rates were declining at 1.5% while Norwegian only 0.9%. In the 1980s we observed some temporal increases in mortality at young adult ages but the general pattern is largely the same. In the

1990s and in the first years of the next century, the progress against mortality again accelerated except for some increases in mortality among young Norwegian males and among the very old in both countries. Death rates were declining on average at 2.7% per annum in Finland, and 2.1% in Norway over the last period.

Among Finnish and Norwegian females the predominantly period pattern is even more pronounced. As for males, the largest progress against mortality was made during 1950s, 1970s and 1990s while in the 1960s and 1980s recession and temporary mortality increase in selected age groups took place. Generally female death rates declined faster and if increased they increased not so as fast as for males. In Finland, for example, during the 1960s average rate of improvement for males was about 0.6% while for females 2.1%; in the 1970s the numbers are 2.5% and 3.4%, respectively.

Compared to Finland and Norway the Swedish female pattern of mortality improvement is characterized by higher stability. This pattern did undergo notable changes in the 1960s when age specific pattern observed in the 1950s was significantly modified. In the 1960s mortality decline accelerated at adult and senior ages while decline at young adult ages slowed down significantly. Starting with the 1970s we observe a persistent decline in death rates with nearly invariable age specific pattern. This is the main difference of the Swedish pattern from that in other Nordic countries. Pattern of improvement in Swedish males is discussed below—there are some distinct features of this pattern which allows us to classify it differently from females.

In Spain very strong progress against mortality took place in the 1950s, for both sexes, and especially at ages below 50. Over this decade death rates in this age group had being falling at a rate 8.5% for males, and at 9.5% for females. The rate of improvement dropped significantly in the 1960s (1.2% for males, and 2.4% for females, all ages) but the progress resumed again in the 1970s (1.7% and 2.8, respectively). The age-specific structure of mortality decline underwent significant changes in the 1970s. For females, for example, the highest age of improvement are found at ages below 5 and in age group 25–80; for males three peaks are observed: in infancy, 35–40 and 65–70. In the 1980s death rates at young adult ages (20–40) temporally increased which soon were succeeded by declines similar in magnitude over the 1990s. At other ages we observe some slow downs in mortality decline as compared with that in the 1970s but persistent decline was uninterrupted over the 1980s and 1990s. In the earlier

2000s there are some signs of acceleration in mortality decline at older ages (70–80) similar to that found in the 1970s.

Data for former East Germany demonstrate plasticity of mortality trends. Before 1990, persistent mortality declines in the male population are observed only in childhood and infancy while at other ages death rates were hovering up and down without any stable trend. In the period 1956–1989 and at ages 0–15 death rates were declining on average 2.3% per year while at older ages average rate of improvement was negative: -0.2%. More favorable developments are observed for females: 2% and 0.9%, respectively. After German unification the progress against mortality in former East Germany dramatically accelerated. Male death rates started to decline on average 4.3% per year and female by 5.0%. Such radical change in the trends of death rates had a profound effect on life expectancy at birth: over the 1990s male life expectancy rose at a pace 4.9 years per decade and female at 4.5 years.

Another distinct pattern revealed by Fig. 3 is characterized by a progressive shift of the high rates of mortality improvement from lower to higher ages. It would appropriate to call it *age shifting pattern* of mortality decline as this pattern is produced as an age group with the highest rates of improvement is gradually shifted to the higher ages. This pattern is clearly expressed in dynamics of mortality in female population of Japan so we discuss mortality development in this population in more detail.

In the 1950s age specific pattern of mortality decline resembles an exponential decay with the highest rates of improvement observed in infancy and persistent slackening of improvement on progression to higher ages. As we can see below this is typical of many other countries though magnitude of decline might vary substantially. For Japanese females average rates of improvement were 11.4%, 11.7%, 12.6%, 9.8%, 6.2%, 4.0%, 3.1%, 1.3% and 0.3% in age groups 0–10, 10–20, …, 70–80 and 80+, respectively. Over years mortality decline at younger ages gradually decelerated: for children less than 10 years old the average rate of improvement dropped from 11.4% in the 1950s to 3% in the 1990s. We even observe increasing mortality, first time for the last fifty years, in the late 1990s among those in their twenties as indicated by blue areas in Fig. 3.

At higher ages (>50) mortality evolution took a completely different path. The death rates were first declining at much slower rates as compared with younger ages but over time the decline significantly *accelerated*. The acceleration was not uniform over ages; it occurred rather

in a cohort-like fashion. The high rates of mortality improvement were gradually moving from lower to higher ages. In the 1960s the highest rates (3.2%) are observed among females in their fifties, in the 1970s among those in their 60s (4.5%), in the 1980s among septuagenarians (4%), and in the 1990s among those aged 80 and over (3%). To support this finding we also computed an age with the highest rate of improvement and followed its progression over time: the highest rate of decline in the year 1960 was observed at age 50, in 1970 at age 60, in 1980 at 72 and in 1990 at 81. Such progression of the highest rates of mortality improvement into higher ages is indicative of age shifting pattern of mortality decline. On the Lexis maps this pattern is manifested by areas with the high rates of improvement evolving in a diagonal fashion (Fig. 3 Japan, Females). Due to large number of ages and cohorts involved the period factors are likely playing a major role in forming this pattern similar to the predominately period pattern.

The age shifting pattern is not that unusual—it is also observed in Australia and New Zealand, especially among males. For Australian males this pattern started to emerge in the late sixties after a decade of slow progress and increases in adult mortality. An average rate of improvement for those aged 40 and over in the 1960s was negative, -0.2%, indicating that the death rates were generally increasing (see also blue area in Fig 3). A radical change in the 1970s is characterized by dramatic decline in death rates at an average rate 2.3%. In the following decade the decline is further accelerated; especially high rates of improvement are found among those in their fifties (3.8%). Such high rates of improvement are gradually shifting into the higher ages over time: in the late 1990s mortality was declining fastest among sexagenarians at a rate close to 5%. Australian females exhibit a pattern very similar to that of males. However the female rates are generally higher and reductions in mortality over last decades are larger.

Pattern of mortality improvement in the male population of New Zealand is also in a close proximity with Australian pattern. In the 1950s progress is observed only in childhood; at other ages the death rates are either stagnant or increasing. In the sixties situation with mortality is further worsened. The death rates were increasing virtually at all ages above 20 at a rate close to 1%. Similar to Australia the beginning of the 1970s also appeared as a turning point in this adverse development—adult mortality started to decline with the highest rates of improvement among those aged 30–50 (3.3%). In the following decade the highest rates of improvement shifted to ages 50–70 (2.7%) with further acceleration of decline in the 1990s (3.8%). In the

early 2000s the process of shifting continued with the highest rates observed now among septuagenarians (3.4%).

Significantly weaker association with age shifting pattern of improvement is found in the female population of New Zealand. Pattern of mortality improvement in this population can be equally well classified as a predominately period pattern. It does have a distinct feature—rates of mortality improvement accelerate over time. If an average rate of improvement in the 1960s was 0.1%, in the 1970s it was already 1%, in the 1980s 2.2% and in the 1990s 2.9%. There are also some signs of slow down in progress in the early 2000s.

Less sharply expressed but still in a close agreement with age shifting pattern of mortality decline were temporal developments in Swedish male mortality. Declines in childhood mortality were strong over the entire period. Relatively high rates of improvement in adult mortality in the first half of the 1950s came to a halt in the 1960s with apparent increases in death rates among those aged 30–50. Situation started to recover only in the late 1970s when decline in adult and senior ages accelerated. The highest rates of declines, about 2.5%, were among those in their late 30s or earlier 40s. Over the years these high rates of improvement were gradually moving to the higher ages as one would expect for age shifting pattern of mortality decline. In the 1990s the progress is further accelerated it somewhat slowed down in the early 2000s.

The third pattern of mortality decline is characterized by presence of substantial cohort-like effects in mortality improvements rates. We were prepared to find some cohort effects as they were long observed in demographic data but their magnitude and widespread existence came completely unexpected. The cohort effects appear to be a prominent feature of the Lexis maps in Fig. 3 so we devoted the rest of this section to their description. Visually they appear to be superimposed or blended with other patterns of mortality improvement. One might speculate whether the cohort effects manifest an unusual mortality experience of a particular cohort or groups of cohorts or their existence is caused by combination of period and age factors. We are not trying here to resolve this issue; simply by using term "cohort effect" we refer to the patterns of mortality improvement as discussed below.

Consider, for example, male population of England and Wales. In the 1950s and in the 1960s pattern of mortality improvement is similar to that found in many other countries: general decline in the 1950s with especially rapid improvements at younger ages and deceleration of progress in the 1960s with increases in mortality among aged 15–20 and those in their forties.

Starting with 1970 we observe emergence of a cohort effect: it is manifested by exceptionally high rates of mortality improvement in the cohorts born around 1930. There is an easily traceable diagonal spur in Fig. 3 stretching from age 50 in the 1980 to age 70 in the late 1990s (scale levels 0.0296-0.0392, color) where the rates of improvement on average are 3.5%. This pattern, with some reservations, can be also classified as age shifting pattern of mortality decline but localization of high rates of improvement around a narrow range of cohorts suggests considering it as a cohort effect superimposed over the predominantly period pattern of mortality improvement. We also find a similar cohort effect in the female population but it is shifted in time centering on the cohorts born in 1935, and less expressed, more blurred over the cohorts.

Instead of accelerating mortality decline might slow down or even reverse along the cohort lines. One of the strongest *negative* cohort effects are found in Danish mortality data. In Fig. 3 the cohort effects are exposed by diagonal stretches of negative rates of improvement shown in blue colors. In the male population negative rates of improvement are observed in the cohorts born around 1950. As one can see in Fig. 3 the rates of improvement in these cohorts are consistently negative from age 10 in 1960 and up to the age 40 in the late 1980s. Rates of improvement are also negative in the female cohorts but not as noticeable as for males. More consistent increases in female death rates are observed among cohorts born around year 1920 (see blue area running from age 45 in 1965 to age 70 in 1990, Fig. 3).

Superposition of a negative cohort effect and age shifting pattern of mortality improvements is illustrated by mortality dynamics in the male population of Japan. The pattern of mortality dynamics in males is very close to that in females expect that mortality decline is interrupted by slowing down the progress in the cohorts born in the 1920s. This feature is indicated by a diagonal area with low rates of mortality improvement stretching from age 55 in 1980 to 75 in 1999 (Fig. 3). Rates of improvement in these cohorts are consistently lower and entire pattern of mortality improvement appears as a combination of age shifting pattern of mortality improvement and this negative cohort effect.

Appreciably large cohort effects, similar to that found in England and Wales and Denmark, exist in male populations of Austria, France, Germany (West), Italy, Japan, Netherlands and Switzerland. Surprisingly, the cohort effects are less frequent in the female populations: the rates of mortality improvements are more regular over cohort lines for females than for males. Consider, for example, France. In the 1950s and in the 1960s we observe pattern

of improvement already found in many other populations: high rates of improvement in the 1950s with rapid decline at younger ages, and general slowdown of progress in the 1960s. Starting with the 1970s French mortality evolves according to age shifting pattern of mortality improvement together with persistent progress in childhood. On the top of this general pattern we observe consistently higher rates of mortality improvement in the cohorts born in the mid-1930s. This cohort effect emerges at the beginning of the 1970s among those in their 30s and continues up to the last year with available data. Two blue spots centered at age 18 in the late 1960s and at age 35 in the late 1990s bear some resemblance to the Danish negative cohort effect but in France they appear as two separate period effects. Mortality dynamics among French females is quite similar to that in males except for this cohort effect. Gains in mortality reduction were larger than in males and the age shifting pattern of mortality improvement emerged also in the 1970s is even more pronounced and greater in magnitude.

Pattern of mortality improvement in the Italian males bears a close resemblance to the French pattern. We also observe some slowdown of progress in the 1960s and emergence of the age shifting pattern of mortality improvement in 1970. Different from France the Italian slowdown in the 1960s appears to be concentrated along the cohort lines. There are two blue strips in the male map: one stretching from age 60 in 1950 and up to age 85 in 1975, and another, less blurred and more cohort specific, runs from age 40 in 1960 and to age 55 in 1975. The French positive effect of cohorts born in the mid-1930s is evident in the Italian data as well, but somewhat less sharp, more spread over cohorts. This pattern might be equally well classified as a compactly expressed age shifting pattern of mortality improvement. Pattern of mortality improvement among Italian females is strikingly close to that of French. The typical pattern of the 1950s with high rates of progress at younger ages was also succeeded in the 1970s by the age shifting pattern of mortality decline. The principle change took place in the 1960s with general slowdown of improvement at younger ages, and some acceleration of the progress at adult ages. Increases in French female mortality in the 1960s are not visible on the Italian map but some deceleration of improvement is evident.

The male cohort effect found in Italian and French data is also a prominent feature of Dutch mortality. As in France this cohort effect comes into a view in the beginning of the 1970s lasting up to the right edge of the Lexis map. Generally, Dutch mortality dynamics was similar to the French except for several important differences. First, the rates of improvement are

smaller in magnitude and the age shifting pattern expressed in the male mortality only. Female mortality followed the predominantly period pattern of mortality improvement with high rates of decline in the 1950s and in the 1970s, and with low progress in the 1960s and in the late 1980s. Age specific pattern of mortality decline also underwent substantial changes over time. Second, developments in death rates among the very old were not as favorable as in France. Rates of improvement were generally lower and persistent mortality decline is frequently interrupted by temporal periods with increasing death rates.

In the late 1950s West German death rates were generally declining. In the 1960s the decline nearly halted and mortality started to increase in several age groups. Over this period average rate of improvement was only 0.27% in the male population supported mostly by reduction in childhood and infant mortality. Starting with 1970 we observe persistent and strong decline in death rates. Especially rapid progress is observed at younger ages with peak in the 1980s. At higher ages the entire surface of German mortality improvement appears if there are two cohort effects operating: consistently higher rates of decline are observed in the cohorts born in the mid-1930s and in the cohorts born around 1910. This pattern is strikingly similar to the French male pattern except that high rates of improvement in generation born around 1910 were not classified as a cohort effect in France due to their spread over age. Instead they were described as an element of age shifting pattern of mortality improvement. We did not find strong cohort effects in the female pattern of mortality improvement; German female pattern matches closely the predominately period pattern of mortality decline. The low rates of the 1960s were succeeded by rapid decline in the late 1970s and 1980s with some slowdown in the progress in the 1990s. Compared to other countries Germany mortality improvement was more strong and persistent over last three decades.

Patterns of mortality improvement in Austria are very close to that found in Germany: small progress in the 1960s with acceleration of mortality decline in the 1970s and persistent progress over the following decades. Unlike Germany and France there is no male cohort effect of the 1930s; we do observe the cohort effect of 1910 as in Germany but it is spread wider over cohort lines making it similar in appearance to a compressed age shifting pattern of mortality decline. Similarities among females are also striking. Progress against mortality at adult and senior ages in Austria was even more persistent and profound than in Germany.

Swiss male pattern of mortality improvement appears unusually regular over time. We do observe acceleration of mortality decline at ages 60+ in the 1970s, temporal increases in young people mortality in the 1970s and 1980s succeeded by the strong decline in the 1990s but compared to other countries the pattern is more stable. Similar to France, Germany and the Netherlands rates of improvement of the cohorts born in the early 1930s are elevated but over time this cohort effect becomes stronger and wider bearing a close resemblance to age shifting pattern of mortality improvement. Unlike Germany and Austria no cohort effect of 1910 is found. Female pattern of mortality improvement matches prerequisites of predominately period pattern of mortality decline. There are three periods with distinct age specific schedules of improvement. Pattern of mortality improvement observed in the 1950s, with the highest rates at younger ages and a little progress at higher, is typical of other countries. Over the following three decades the rates of decline at adult and senior ages significantly accelerated while the progress at young ages slowed down with evident temporal increases in mortality of young persons (see blue areas in Fig. 3). As a result age-specific pattern of mortality decline underwent principal changes since 1960—age profile of decline is not gradually declining from the highest rates in childhood to the low rates at senior and old ages as in the 1950s but there are two age groups (<10, 60–80) where death rates decline fastest (Fig. 3). In the 1990s decline at adult and senior ages slowed down with simultaneous acceleration of decline at younger ages—we observe a reversal of age specific pattern to that of the 1950s.

Implications for Mortality Forecasting

In this section we are going to demonstrate how field of mortality forecasting can benefit from exploration of patterns of mortality improvement by smoothing and visualization techniques presented here. Most of the existing models for forecasting mortality are of extrapolative nature—provided a matrix of death rates the goal is to extrapolate such matrix into the future in a reasonable way. One of the simplest and widely used methods is extrapolation of log-linear trend in death rates. Death rate at each age or age group is assumed to decline on its own rate and by computing rates of decline over some recent period forecasts of future death rates can be produced. All aggregate quantities of future life tables such as life expectancy can be computed from the forecasted matrix of death rates.

As it clear from the design of this method it is based on two main assumptions. First, pattern of mortality improvement is assumed to remain unchanged over the period of forecast. Second, pattern of mortality improvement is assumed to be stable over the period employed for estimating rates of decline. In terms of Lexis maps the pattern shown in Fig. 2 b) is assumed (for a particular population age specific profile might be different but invariability in time direction retains).

How exploration of historical mortality improvement (Fig. 3) might benefit a forecaster? First, pattern of mortality decline might provide a better idea about which period should be selected for computing average rates of improvement. It is better to maximize the period to obtain more stable estimates of rates of decline but the pattern of improvement should not change much over the selected period. Second, we might get a better view how assumptions about future correspond to the historical trends. Finally, it is important to analyze our expectations about future in context of development in other countries. Here, exploratory analysis of mortality improvement patterns in other populations might help to refine assumptions about future.

To be more specific let us look at mortality developments in female population of United States. Suppose that we need to build a forecast of mortality for next 10 or 20 years. By exploring pattern of mortality improvement provided in Fig. 3 we observe that it was relatively stable over last two decades (1980–2000). It would be reasonable then to build our forecast based on average rates of decline estimated over this period.

Alternatively, we can pose another question—how reasonable would be to expect a structural shift in mortality improvement in the coming years? Should we expect that mortality decline in this population will slow down (as in the 1960s) or rather accelerate over time as in the 1970s? To answer this question we need to step out from analyzing mortality in a single population and look at it in context of developments in other countries.

Compared to other countries a little progress has been done in United States in reducing mortality at ages above 60 (Fig. 3). From 1980 to 2000 the U.S. female life expectancy at age 60 rose only by 1.02 years while life expectancies in Canada, Sweden and Japan by 1.84, 2.22 and 5 years, respectively. It is clear from these numbers and from Fig. 3 that progress in reducing U.S. mortality was atypical of that in other developed countries. Provided that the health issues of adult and senior females are prioritized and by building on experience of other countries it is

very possible to influence course of mortality in a favorable way. Acceleration of mortality decline in the coming years is therefore a plausible scenario for building this mortality forecast.

More sophisticated methods of mortality forecast aim at modeling entire matrix of death rates in attempt to capture structure of death rates by several time-varying parameters (McNown and Rogers 1989), (Lee and Carter 1992). In the Lee-Carter model (this method gained certain popularity in the last years) death rates are assumed to have the following structure:

$$\ln m_{xt} = a_x + b_x k_t \tag{5}$$

where m_{xt} is observed age specific death rate, a_x, b_x are age specific parameters independent of time, and k_t is time specific parameter independent of age. Applications of this model to data for many developed countries suggest that the k_t index follows a simple linear trend (Lee and Carter 1992), (Tuljapurkar; Li, and Boe 2000). In this case mortality forecasting can be carried out by forecasting k_t by random walk with drift process assuming that a_x and b_x are invariant over time.

What surface of mortality improvement is implied by this model? By taking a derivative

$$r_{xt} = -\frac{d}{dt}\ln m_{xt} = -b_x k_t' \tag{6}$$

one can see that rate of mortality improvement in a given year is a product of the b_x schedule and the negative derivative of the k_t. If k_t is a linear function—as it is assumed by random walk with drift process—then k_t' is constant and r_{xt} is also constant over time implying that each age-specific death rate declines at its own rate. We discussed such case before and provided an example of such surface in Fig. 2.

Beyond this pattern of improvement this model is also capable of capturing periods of stagnation, acceleration and deceleration in mortality decline. If k_t is constant over some period then k_t' is zero and the rate of improvement r_{xt} is also zero implying that no changes in death rates take place over this period. If k_t declines faster than a linear function this implies that mortality decline accelerates, and if k_t declines slower it slows down. Moreover, if k_t is increasing this implies that death rates are also increasing (we assume here that $b_x > 0$).

The model appears to handle a large variety of patterns of mortality improvement but it highly restricts the space of age specific profile that the rates of improvement might take. From (6) follows that the age profiles are proportionally related:

$$r_{xt_2} = \frac{k'_{t_2}}{k'_{t_1}} r_{xt_1} \tag{7}$$

and, consequently, proportional to the schedule b_x. As a result one should not expect a good fit if the age specific profile of mortality decline goes through structural changes over time.

Exploratory analysis of mortality improvement surfaces might provide useful insights if the age specific profile underwent any structural changes, and pinpoint periods of stable reduction in death rates. Let us consider female population of United States. The predominately period of mortality improvement assigned to this country is expected to be handled well by the Lee-Carter model: the periods of acceleration (the 1950s and the 1970s) will correspond to steep slopes on the k_t vector and the periods of slowdowns (the 1960s, 1980s and 1990s) to gradual, slowly changing values of k_t. We can also see from Fig. 3 that age specific pattern of mortality improvement was not stable over time: in the 1950s, for example, the highest rates of improvement are observed at ages 20–30 while in the 1970s the highest rates are in infancy, at ages 30–40 and at ages 70–90. Due to its structural constraints the model will not captures such changes in the age profiles rather it will further average the rates given in Fig. 3. Temporal increases in death rates in selected age groups as depicted by blues colors in Fig. 3 are also ignored by the model.

The Lee-Carter procedure might still be successful in long term forecast if the predominantly period pattern of mortality decline will prevail in the future provided that the changes in age specific profile are not outstanding. One might not be able to predict temporal increases in death rates (see also blue areas in Fig 3), and one might underestimate death rates during periods of recessions and overestimate during periods of acceleration but on average the performance might be acceptable.

Performance of this model might significantly worsen if structural changes in the age profile of mortality improvement are profound and persistent. Let us consider application of the Lee-Carter model to the female of population of Japan. As it follows from Fig. 3 the age

shifting pattern of mortality decline was emerging in the 1960s replacing typical pattern of mortality improvement of the 1950s. To find out a magnitude of bias expected by failure to anticipate structural changes in mortality improvement we fitted the Lee-Carter model to Japanese female data for the period 1950-1969 and produced forecasts up to the year 2000. The fitting procedure as in the original article of (Lee and Carter 1992) with k_t parameter adjusted to the total number of deaths with jump off correction as suggested by (Lee and Miller 2001). Percent of explained variance (97.5%) by the first term of SVD indicates a good fit (see also discussion below about this measure) and trend in the k_t is highly linear (OLS fit, $R^2 = 0.98$ assuring that we can proceed with producing forecasts in a usual way.

Forecasted and observed life expectancies for the period 1970–1999 are compared in Fig. 4. The observed life expectancy— both at birth and at age 65— is increasing faster than that predicted by the Lee-Carter procedure. In the last years the observed life expectancy at birth is about 2 years higher than the forecast, and at age 65 is about 3.4 years higher. Even more remarkable differences are found at age 80 (not shown) where the difference between life expectancies is 2.8 years. Main reason for such bias is that age shifting pattern of mortality improvement is not anticipated by the Lee-Carter procedure. The procedure assumes that rates of mortality decline in the period 1950–69 at higher ages are unchanged while in reality the decline is accelerated. Failure to account for such acceleration results in underestimating gains in life expectancy.

It is important to understand that age shifting pattern of mortality decline can not be well approximated either by the method assuming constant rates of mortality decline or by the Lee-Carter procedure. It might be obvious in the former case but less clear in the latter. The Lee-Carter procedure includes many parameters and the common goodness of fit methods might produce an impression that the model fits data very well capturing all aspects of the data. To demonstrate it we fitted again the Lee-Carter model to the female population of Japan but now for the entire range of data: 1950–1999. The percent of variation explained by the first SVD term is 96.8%—a high value indicating a good fit. The trend in k_t (Fig. 5 a)) is also highly linear (OLS $R^2 = 0.9955$). What would an indication of a lack of fit? First, we investigated residuals defined as $\delta_{xt} = \ln m_{xt} - \ln \hat{m}_{xt}$. In this definition the residuals are simply ratios of observed and fitted death rates. Clear and revealing display of residuals is obtained if they are

arranged in the year by age matrix and displayed as a Lexis map. Fig. 5 b) shows the Lexis map of residuals (the data are unsmoothed). As before $\delta_{xt} > 0$ are shown in magenta and $\delta_{xt} < 0$ in blue with scale levels indicating magnitude of residual value. If the model captures data well then the residual pattern should be close to random but this is obviously not the case here. More insights in the goodness of fit can be provided by analyzing surface of mortality improvement implied by the *fitted* death rates $\hat{m}_{xt}$. We used the same smoothing for observed and fitted death rates and plotted the resulting Lexis maps of mortality improvement in panels c) and d) of Fig. 5. The age shifting pattern of mortality decline apparent in Fig. 5 c) completely disappears in Fig. 5 d). This result provides an additional support that the model does not fit data very well.

The analysis conducted here clearly demonstrates that the age shifting pattern of mortality decline requires new models of mortality forecasting other than assuming that mortality improvement rates are constant or by adopting the Lee-Carter approach. This pattern of mortality improvement implies accelerated progress in reduction of mortality at advanced ages which might provide an additional boost to sustain growth in life expectancy. Assuming, explicitly or implicitly, that life expectancy will level off over time might be inconsistent with this pattern of mortality improvement due to propagation of high rates of declines into the higher ages. Existence of such pattern of improvement in historical mortality data makes it an eligible candidate for future scenarios of mortality dynamics.

Another important empirical feature of mortality improvement surfaces is existence of appreciable cohort effects. As it is revealed in Fig. 3 such cohort effects exist in many countries and due to their magnitude they cannot be regarded as temporal unimportant fluctuations. Forecasting models which explicitly take such cohort effects into account are also underdeveloped. This finding of a particular importance for the short term forecasts where the cohort effects can be taken into account assuming that the recently observed patterns extend into future. For long term forecasts it would be quite difficult to predict emergence and magnitude of such cohort effects as not much is known about their etiology. Obscure origins of the cohort effects require further work so plausible explanations of the observed patterns could be provided.

Conclusions

Patterns of mortality improvement over age and time in national populations can be well revealed by applying tensor product spline for smoothing raw rates of improvement with subsequent visualization of results with demographic Lexis maps. Application of the proposed method to the data for 18 countries since 1950 suggests that patterns of mortality improvement varied significantly between populations. It is possible nevertheless to highlight some common features and provide a broad classification of the observed patterns.

In the 1950s age specific pattern of mortality improvement was remarkably similar among countries. The highest rates of improvement are observed in infancy and childhood with progressive slackening of rates of improvement at the higher ages. This pattern underwent substantial country-specific transformations in its shape over time. The transformation took place mostly in the 1960s when death rates for many countries were stagnant or increasing. Starting with the 1970s mortality decline took different paths in different countries which could be broadly classified as predominantly period and age shifting patterns of mortality decline. In the first case the rates of improvement remain stable over certain, usually about 10 years, period of time with possible change in age shape between periods. The age shifting pattern is characterized by progressive shifting of high rates of improvement into the higher ages.

Another important observation is widespread cohort-like patterns in mortality improvement rates. Such cohort effects are usually superimposed on an underlying pattern of mortality improvement. The origins of such cohort effects are largely unknown. They might be a manifestation of mortality experience of individual generations or they might a variant of age shifting pattern of mortality decline concentrated along cohort lines but governed mostly by the period factors. They appear also more frequently in the male populations and several European countries share timing of such effects.

We also observe that geographically close countries e.g. United States and Canada, Australia and New Zealand share more similarities in their patterns of mortality decline than that far apart. Even if mortality levels might be significantly different between countries the temporal mortality dynamics in individual age groups appears to be highly correlated.

The findings presented here have a potential importance for building better models of mortality forecasts. First, the techniques we used provide indispensable insights in the data on a

stage of exploratory data analysis. Second, they can facilitate selection an appropriate forecasting method or parameters (e.g. length of death rate series) of a specific method. Finally, they can enrich goodness of fit methods providing an excellent residual display or insights into mortality improvement pattern implied by a specific model.

We also note that there is a lack of models explicitly incorporating the observed cohort effects or the age shifting patterns of mortality improvement. The latter of a particular importance for the field of forecasting as accelerated decline of death rates at high ages and propagation of high rates of improvement into the very old ages might sustain a persistent growth in life expectancy while virtually all currently existing methods assume progressive slowdowns and leveling off in life expectancy over time (for exceptions see (White 2002) and (Oeppen and Vaupel 2002)). The prevailing beliefs might prove wrong as they are inconsistent with the accelerated progress against mortality implied by the age shifting pattern of mortality improvement. The linear increase in the best practice life expectancy as observed by (Oeppen and Vaupel 2002) might serve as alternative and more plausible basis for developing future models of forecasting.

References

1. Australian Demographic Databank. Australian Centre for Population Research, Research School of Social Sciences, The Australian National University (http://acpr.edu.au).

2. de Boor, Carl. A practical guide to splines. Revised Edition, Applied Mathematical Sciences Vol 27. New York: Springer; 2001; ISBN: 0-387-95366-3.

3. Dierckx, Paul. Curve and Surface Fitting with Splines. United States: Oxford University Press Inc., New York; 1993; ISBN: 0-19-853441-8.

4. Diggle, Peter J. Time Series. A Biostatistical Introduction. Oxford University Press; 2000.

5. Human Mortality Database. Available online at www.mortality.org: University of California, Berkeley, United States and Max Planck Institute for Demographic Research, Germany.; 2004.

6. Lee, Ronald D. and Carter, Lawrence R. Modeling and Forecasting U.S. Mortality. Journal of the American Statistical Association. 1992 Sep; 87(419):659-675.

7. Lee, Ronald D. and Miller, Timothy. Evaluating the Performance of the Lee-Carter Method for Forecasting Mortality. Demography. 2001 Nov 4; 38.

8. London, Dick. Graduation: The Revision of Estimates. Winsted, Connecticut: ACTEX Publications; 1985.

9. McNown, R. and Rogers, A. Forecasting Mortality: A Parameterized Time Series Approach. Demography. 1989 Nov 4; 26(4):645-660.

10. Oeppen, Jim and Vaupel, James W. Broken Limits to Life Expectancy. Science. 2002 May 10; 296.

11. Schoenberg, I. J. Spline Functions and the Problem of Graduation . Proc. Amer. Math. Soc. 1964; 52.

12. Tuljapurkar, Shripad; Li, Nan, and Boe, Carl. A universal pattern of mortality decline in the G7 countries. Nature. 2000 Jun 15; 405.

13. Vaupel, J. W.; Wang, Zhenglian.; Andreev, Kirill F., and Yashin, A. I. Population Data at Glance: Shaded Contour Maps of Demographic Surfaces over Age and Time. Odense University, Denmark: Odense University Press; 1998; ISBN: 87-7838-338-2.

14. White, Kevin M. Longevity Advances in High-Income Countries, 1955-96. Population and Development Review. 2002 Mar; 28(1).

15. Whittaker, E. T. On a New Method of Graduation. Proceedings of the Edinburgh Mathematical Society. 1923; 41:63-75.

16. Wilmoth, John R. Methods Protocol for the Human Mortality Database. Technical Report, Demographic Department, University of California, Berkeley, USA. Available Online at Www.Mortality.Org. 2002.

Figures

Figure 1 Raw Estimate of Surface of Mortality Improvement in the female population of Japan.
This matrix has been produced by applying (1) to the matrix of Japanese death rates

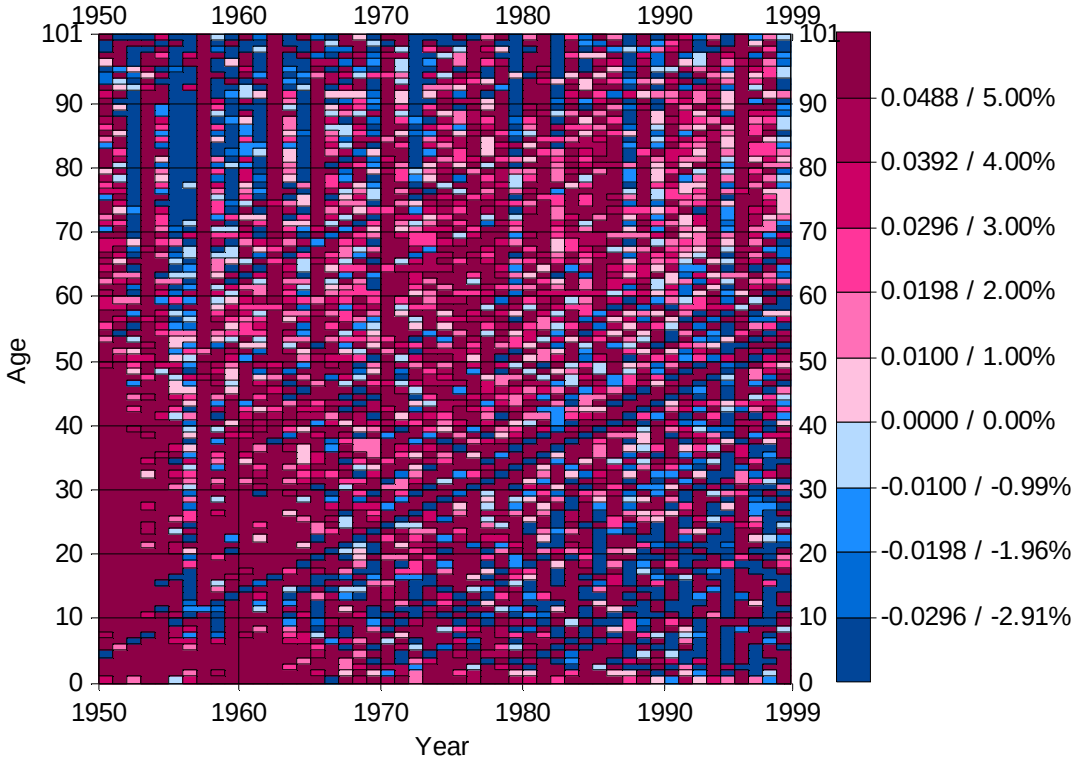
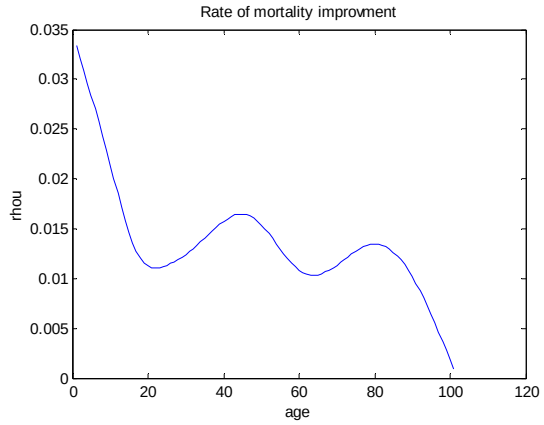
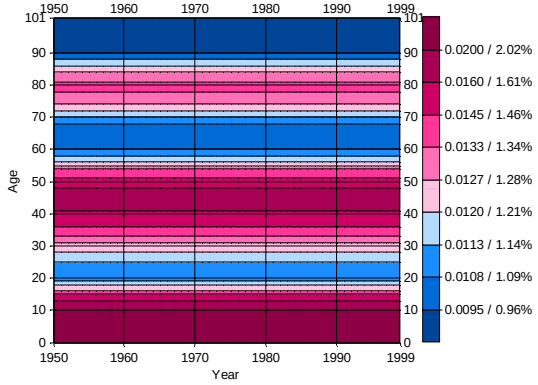


Figure 2 Evaluating performance of the tensor product spline on simulated data

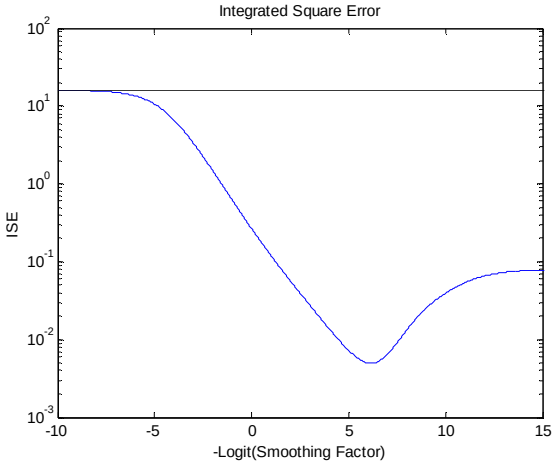
a) Age specific schedule of mortality decline



b) Pattern of mortality improvement



c) Integrated Square Error



d) Smoothing spine with the factor = 0.002

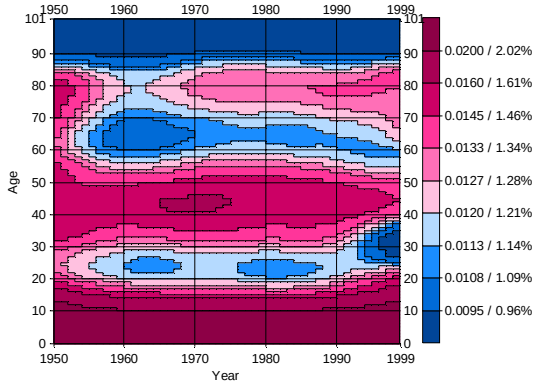
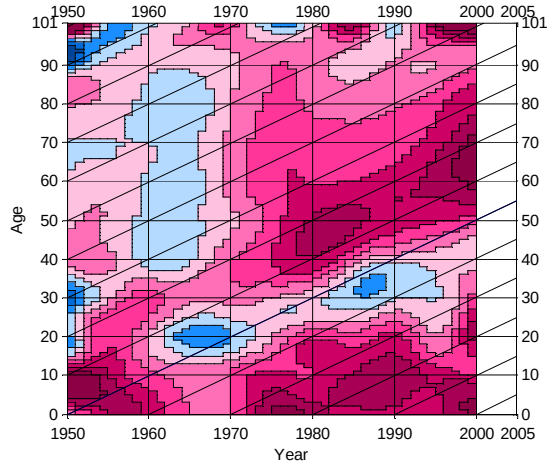
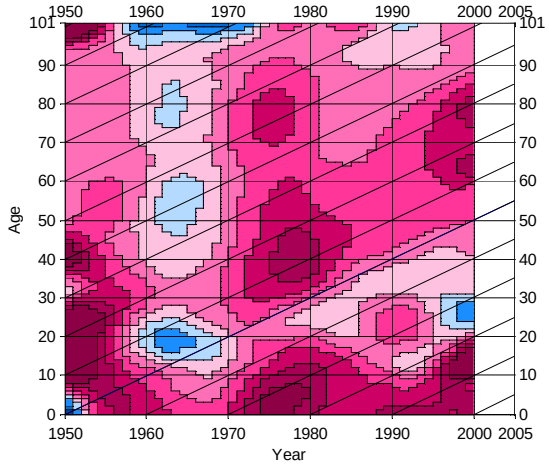


Figure 3 Patterns of Mortality Improvement over Age and Time

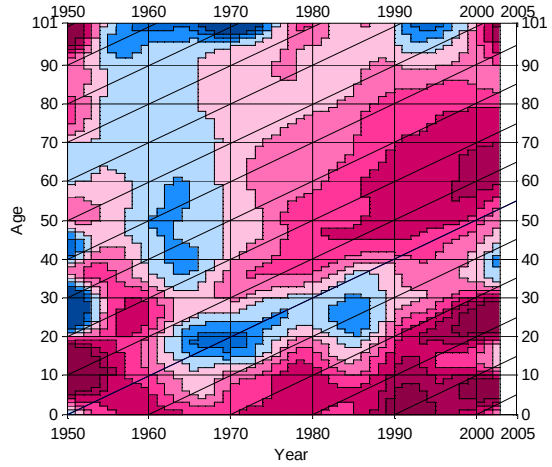
Australia, Males



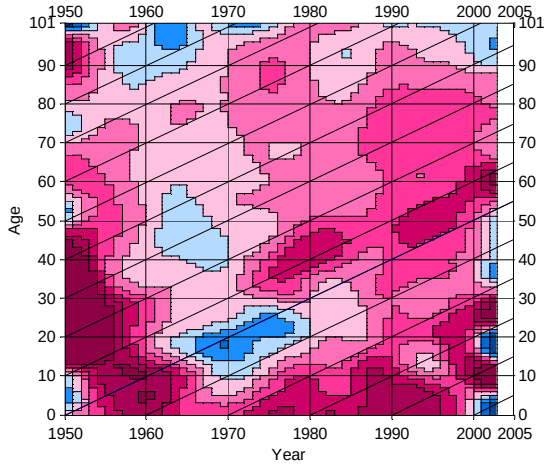
Australia, Females



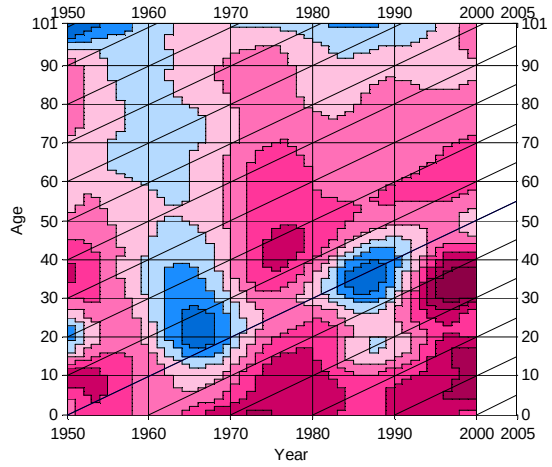
New Zealand, Males



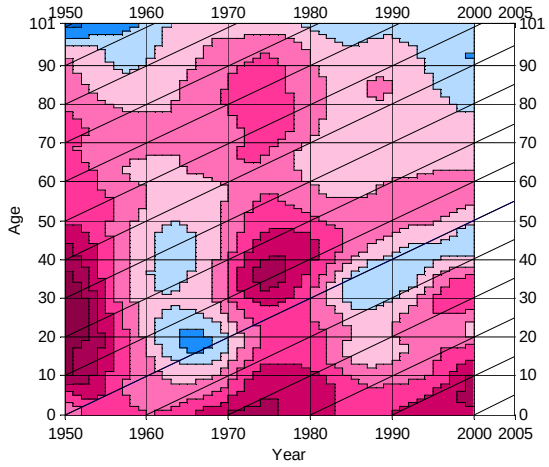
New Zealand, Females



United States, Males



United States, Females



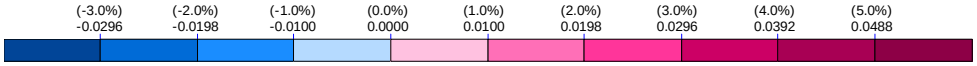
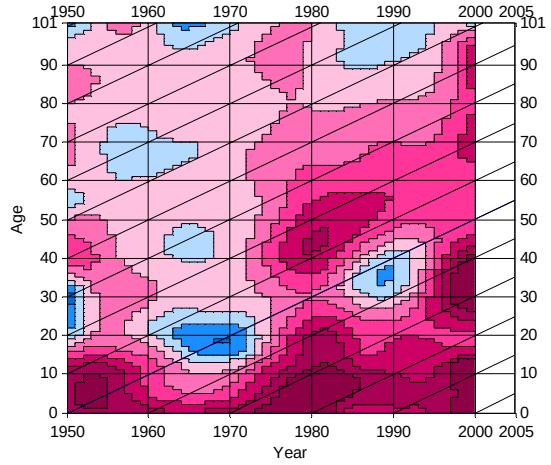
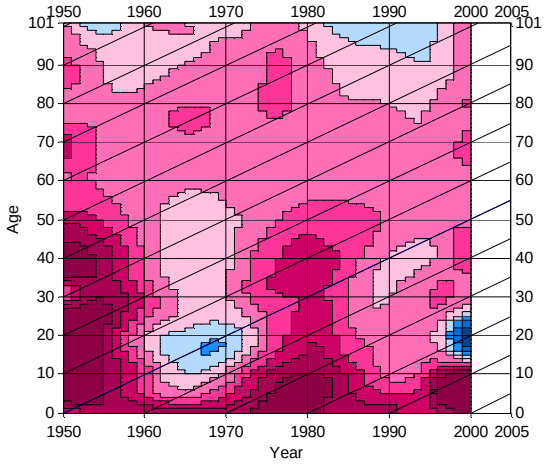


Figure 3 continued

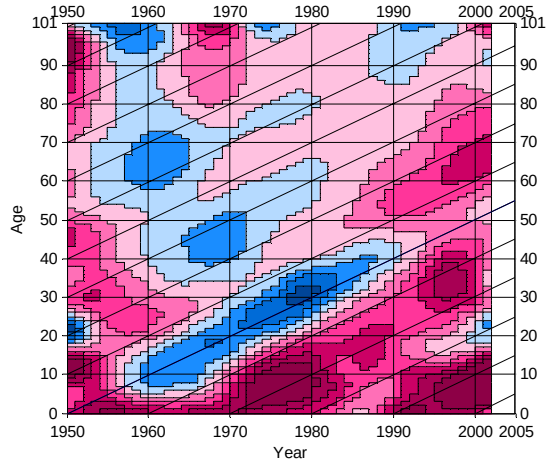
Canada, Males



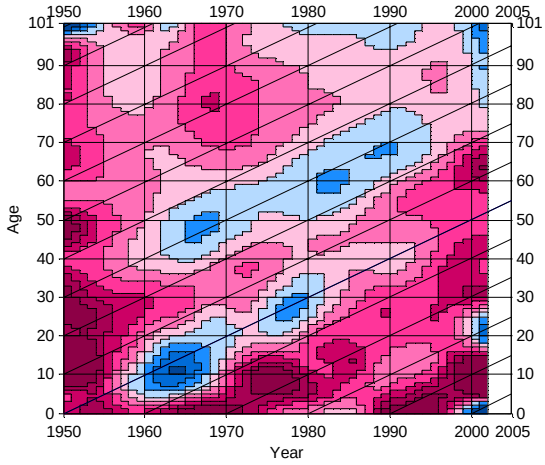
Canada, Females



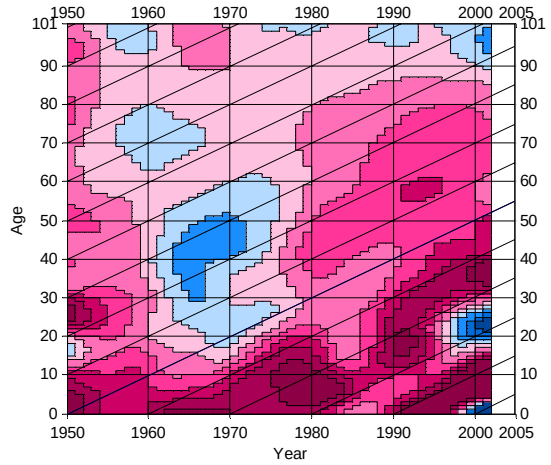
Denmark, Males



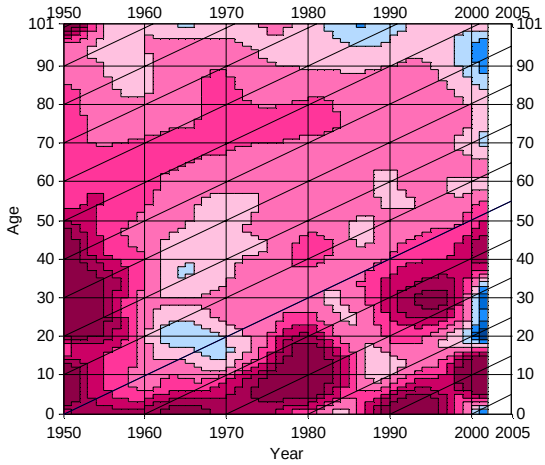
Denmark, Females



Sweden, Males



Sweden, Females



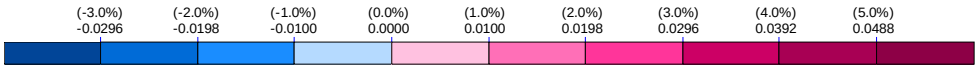
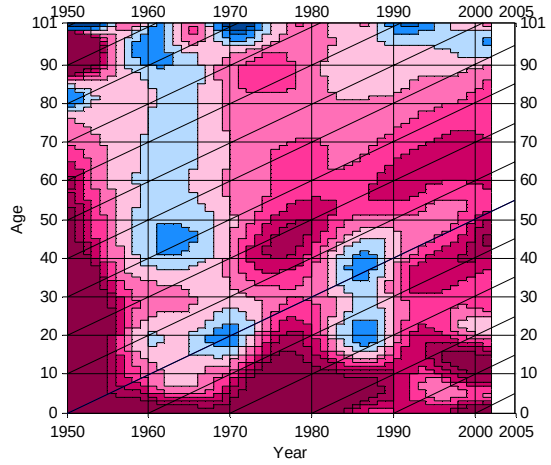
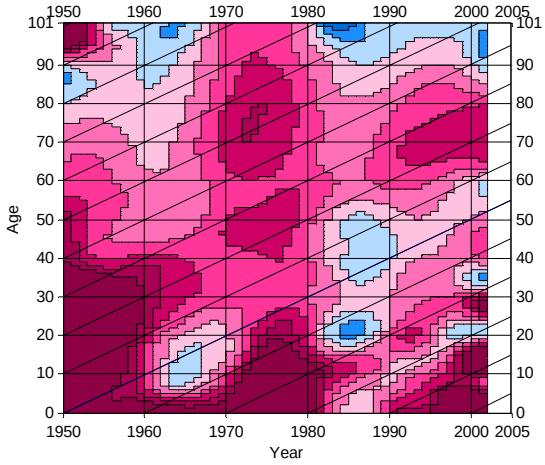


Figure 3 continued

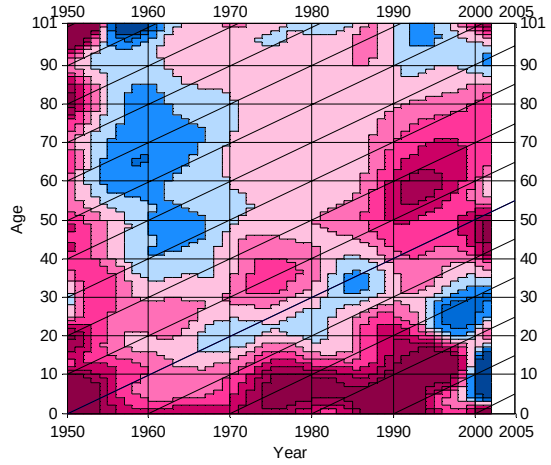
Finland, Males



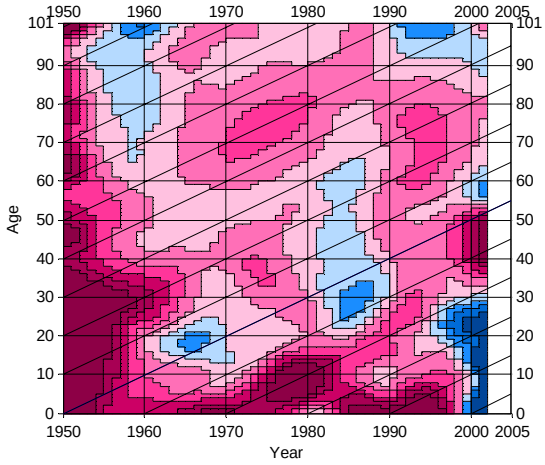
Finland, Females



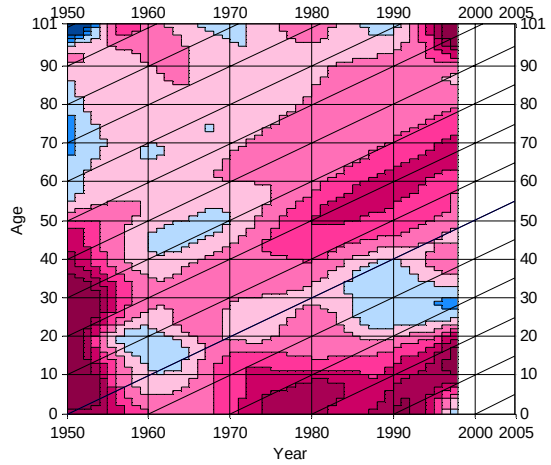
Norway, Males



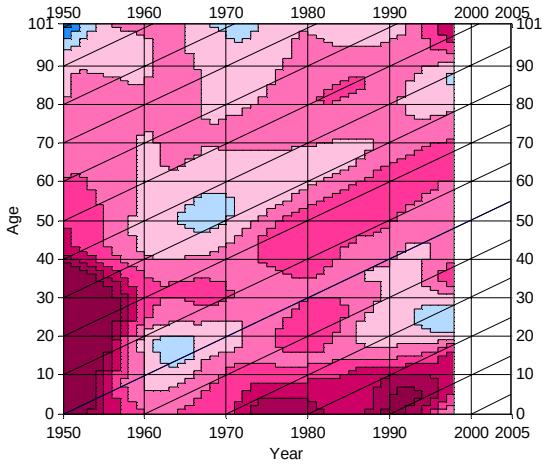
Norway, Females



England & Wales, Males



England & Wales, Females



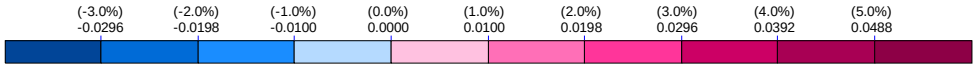
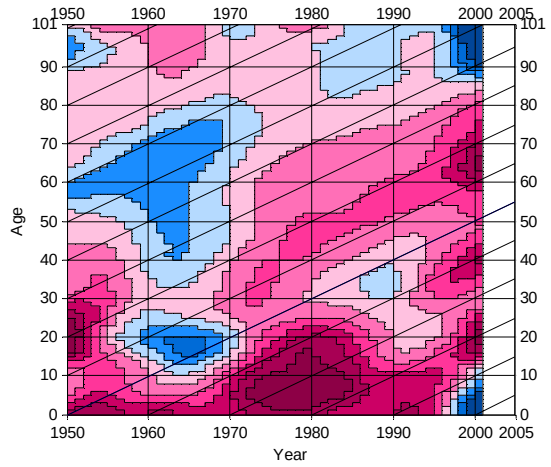
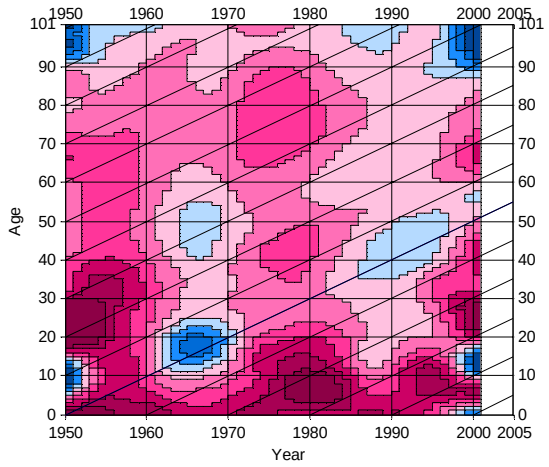


Figure 3 continued

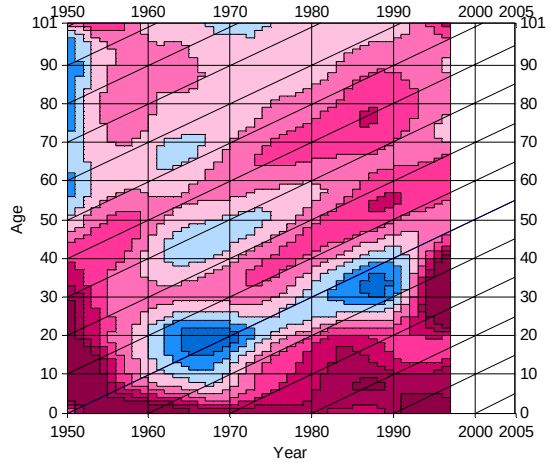
Netherlands, Males



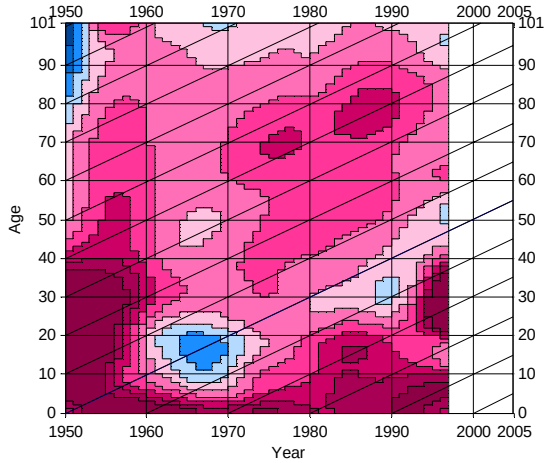
Netherlands, Females



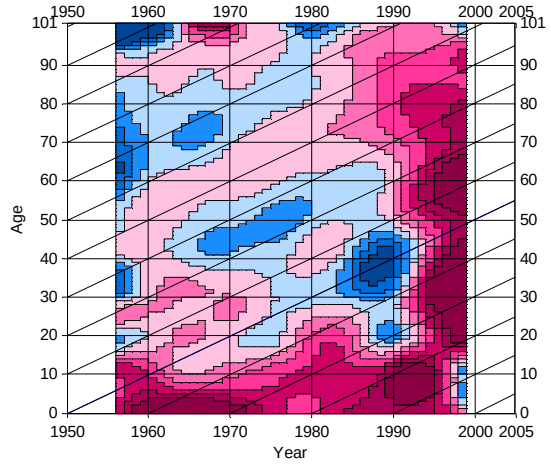
France, Males



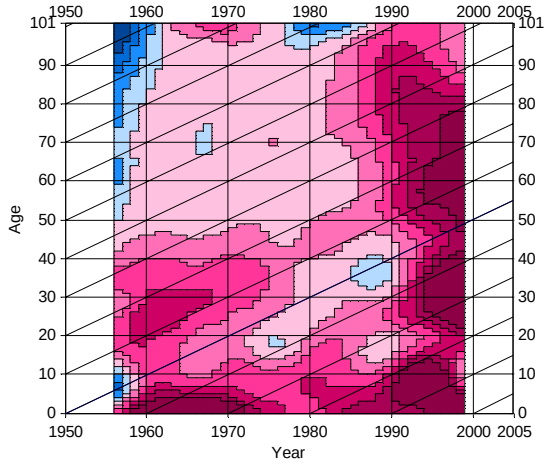
France, Females



Germany (East), Males



Germany (East), Females



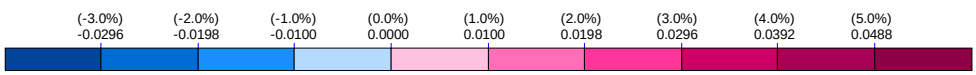
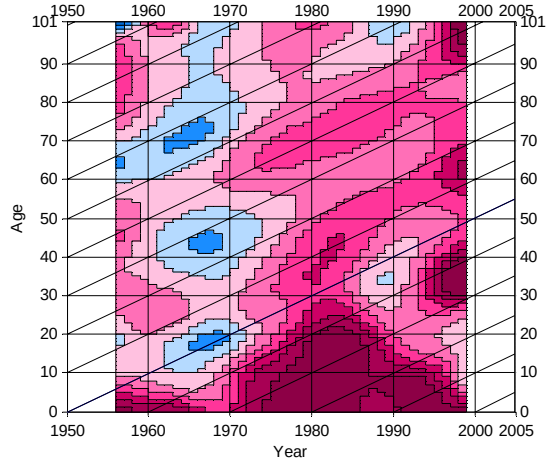
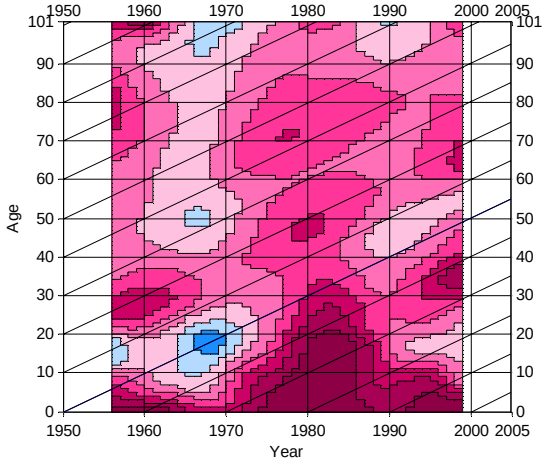


Figure 3 continued

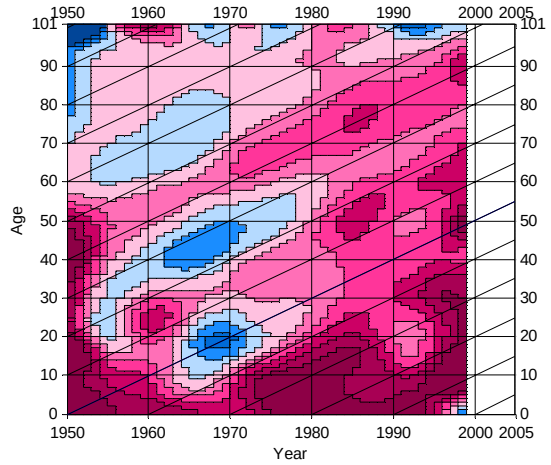
Germany (West), Males



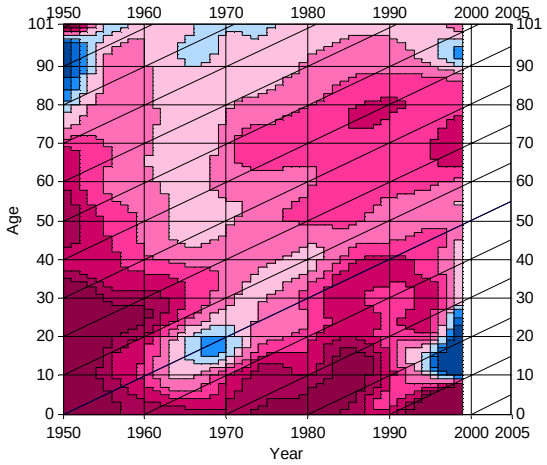
Germany (West), Females



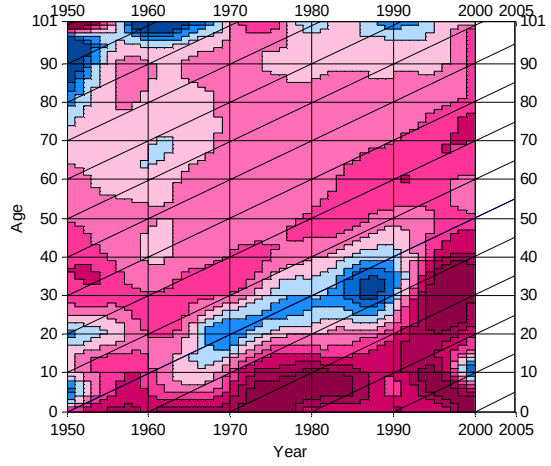
Austria, Males



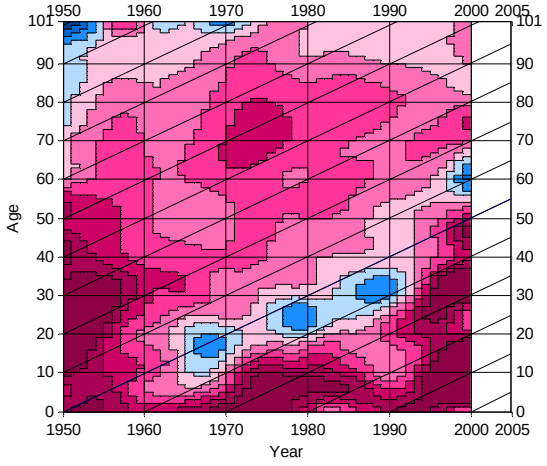
Austria, Females



Switzerland, Males



Switzerland, Females



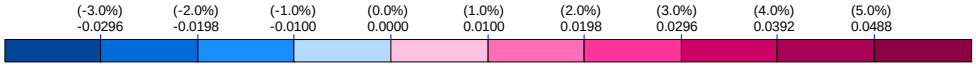
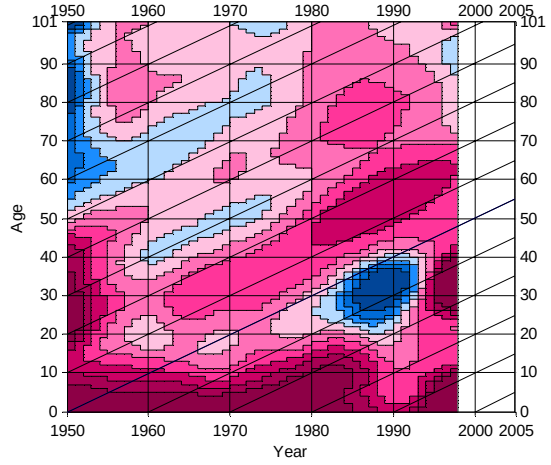
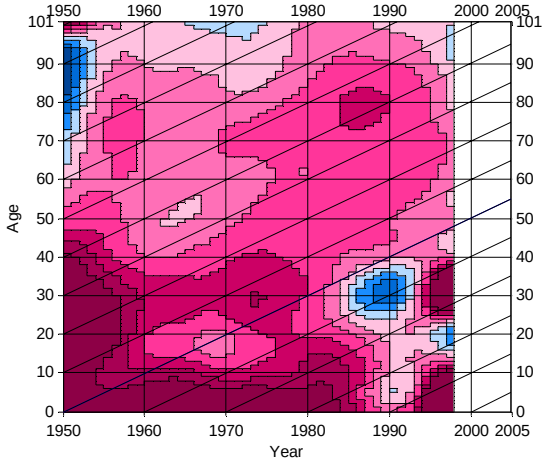


Figure 3 continued

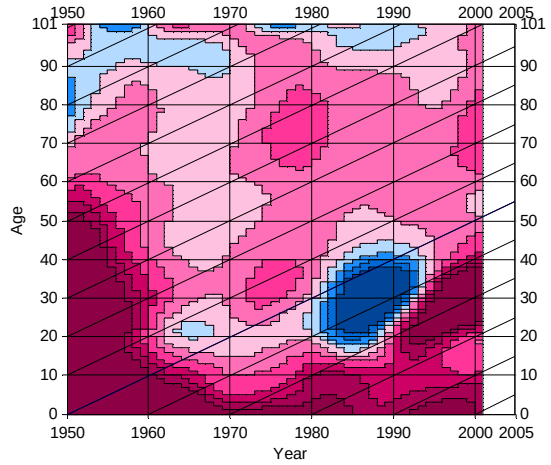
Italy, Males



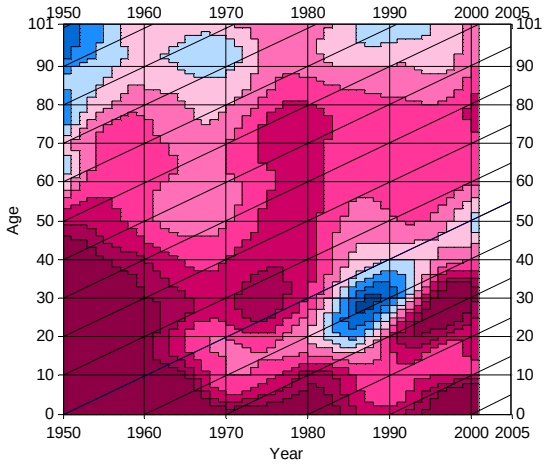
Italy, Females



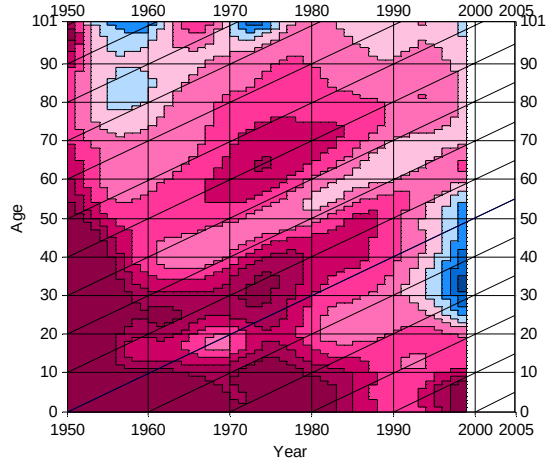
Spain, Males



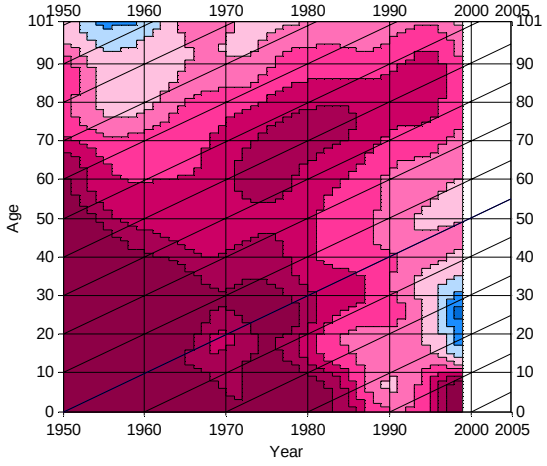
Spain, Females



Japan, Males



Japan, Females



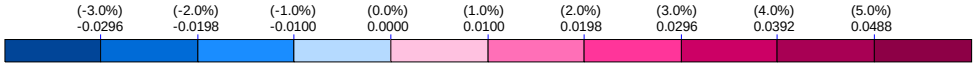


Figure 4 Comparisons of observed life expectancy in Japan, Females with forecasts by the Lee-Carter model fitted to the period 1950–69

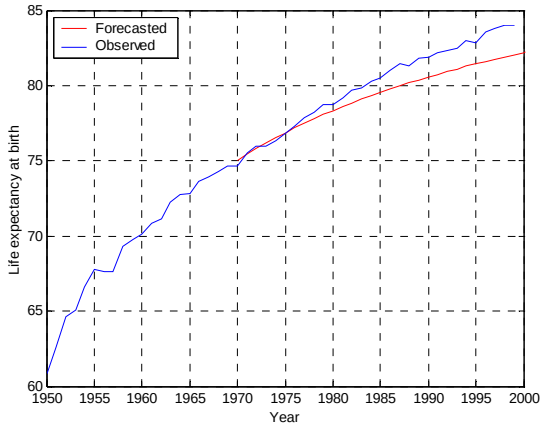
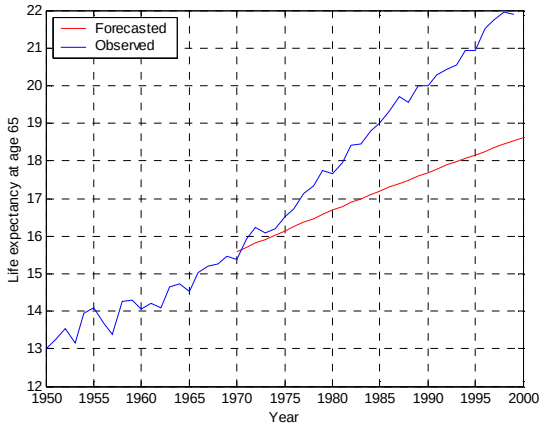
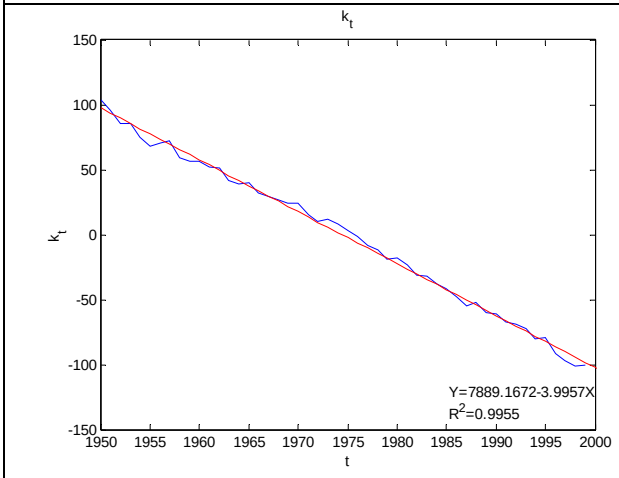
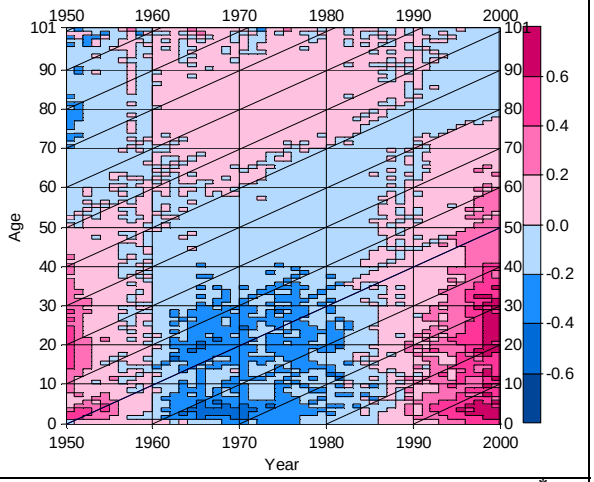
a) at birth	b) at age 65
	

Figure 5 Fit of the Lee-Carter model to the mortality surface of the female population of Japan, 1950–1999

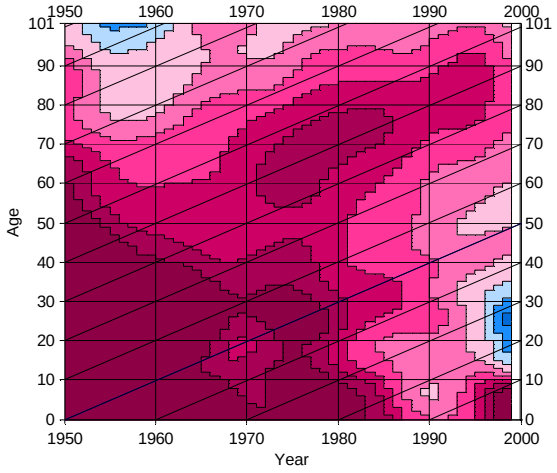
a) Estimate of the k_t parameter



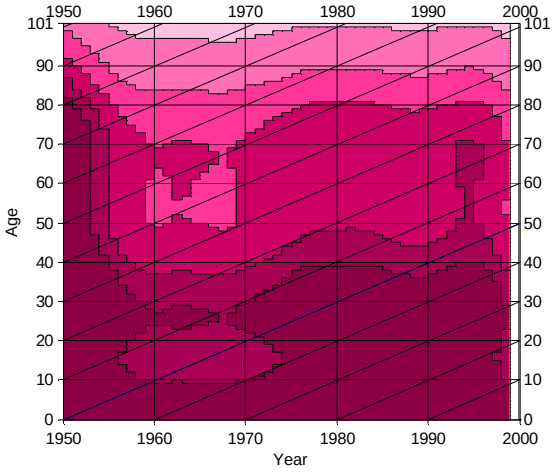
b) Lexis map of residuals, $\ln m_{xt} - \ln \hat{m}_{xt}$

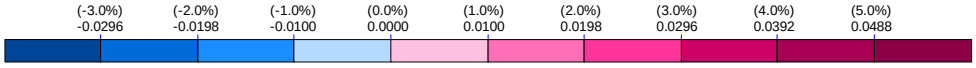


c) Observed surface of mortality improvement*



d) Fitted surface of mortality improvement*





* Both, observed and fitted surfaces of mortality improvement have been smoothed with the tensor product spline, smoothing factor = 0.02