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Marriage and Divorce since World War II: Analyzing the Role of Technological Progress on the Formation of Households

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Abstract

Since World War II there has been: (i) a rise in the fraction of time that married households allocate to market work, (ii) an increase in the rate of divorce, and (iii) a decline in the rate of marriage. What can explain this? It is argued here that technological progress in the household sector has saved on the need for labor at home. This makes it more feasible for singles to maintain their own home, and for married women to work. To address this question, a search model of marriage and divorce is developed. Household production benefits from labor-saving technological progress.

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Subject Area: Macroeconomics

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1 Introduction

Consider the following two facts that have helped reshape U.S. households over the last 50 years:

1. A smaller proportion of the adult population is married now than 50 years ago – Figure 1.[1] Eighty-two percent of the female population in 1950 was married (out of non-widows between the ages of 18 and 64). By 2000 this had declined to 62 percent. Adults now spend a smaller fraction of their lives married.[2] In 1950 females spent about 88 percent of their life married as compared with 60 percent in 1995. Underlying these facts are two factors.

   (a) Between 1950 and 1990, the divorce rate doubled from 11 to 23 divorces per 1,000 married women (between the ages of 18 to 64) – Figure 2.

   (b) At the same time, the marriage rate declined. Exactly how much is somewhat sensitive to the particular age group used for the calculations.[3] In 1950 there were 211 marriages per 1,000 unmarried women as compared with just 82 in 2000 (again, out of non-widows between the ages of 18 to 64).

2. The amount of time allocated to market work by married households has increased

[1] Sources: (1) The marital status of the population is reported in the U.S. Census Bureau publications *Marital Status and Living Arrangements* (March 1950 to March 1998) and *America's Families and Living Arrangements* (March 1999 to March 2000). (2) The fraction of life spent married is from Schoen (1983) and Schoen and Standish (2001). (3) The divorce and marriage rates are contained in Clarke (1995a,b).

[2] The fraction of time spent married is calculated as follows: First, data on life expectancy, e, and the fractions of total life spent as never married, n, married, m, and divorced, d are collected from Schoen (1983) and Schoen and Standish (2001). This data covers each year between 1950 and 1980, and the years 1983, 1988, and 1995. Second, on the basis of these numbers, the figures presented in Figure 1 are then calculated as

$$\frac{em}{en - 18 + em + ed}.$$

[3] The basic problem is that data is not available for marriages by age group. Data is available on the number of unmarried women by age group. Hence, the marriage rate for a particular age group is computed as the total number of marriages divided by the total number of unmarried women in the given age group.

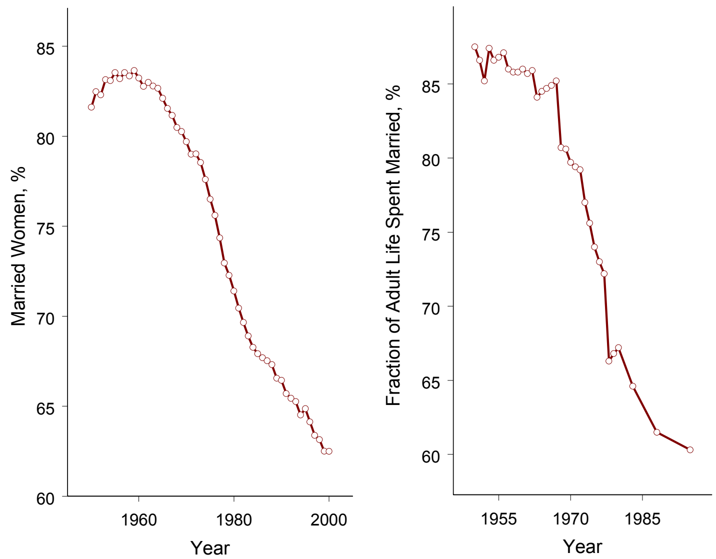

Figure 1: Marriage, 1950-2000.

markedly over the postwar period – see Figure 3.[4] This is mainly due to a rise in labor-force participation by married females. In particular,

(a) In 1950 a married household in the 24-to-54-year-old age group spent 25.5 hours per week per person working in the market. This compared with the 31.3 hours in a single household. Thus, singles worked more in the market on average than did married couples. At the time, only 23.7 percent of married women worked, compared with 78 percent of single ones.

(b) By the year 1990 the labor effort expended per person by married households had risen to 33.5 hours per week. This exceeded the 30.6 hours spent by a single household. Almost as many married females were participating in the labor market (71 percent) as single ones (80 percent).

[4] Source: Simple tabulations based on data extracted from IPMUS-USA, Minnesota Population Center, University of Minnesota. See Section 6.1.1 for more detail.

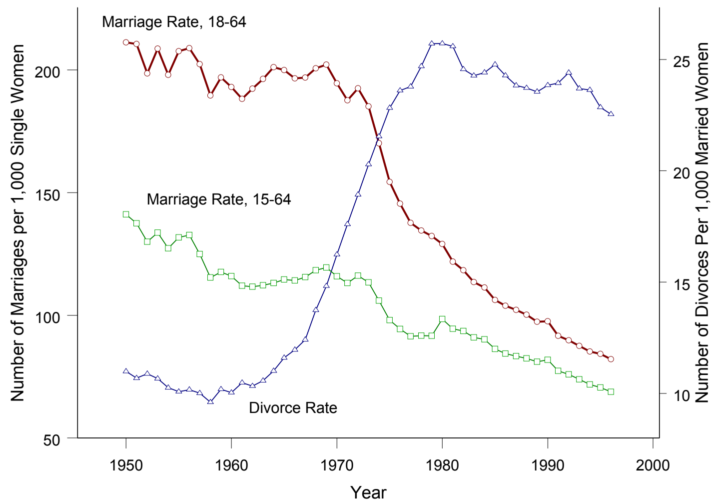

Figure 2: Rates of Marriage and Divorce, 1950-2000.

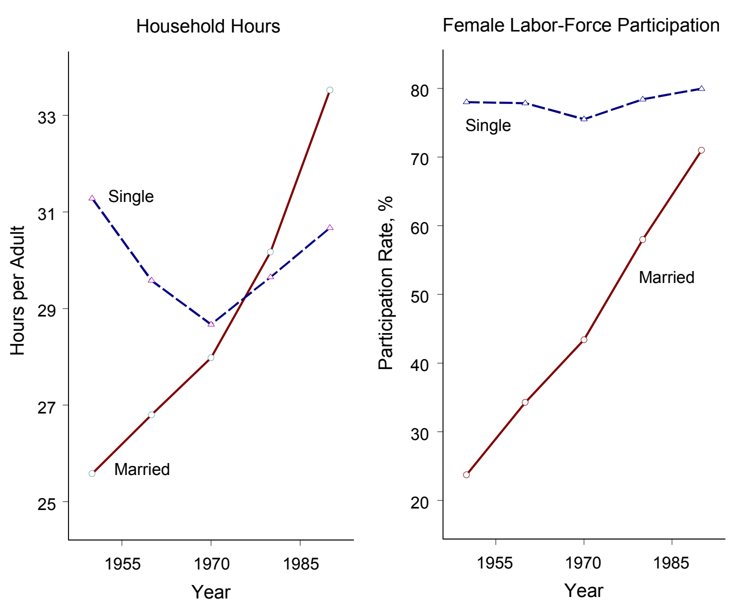

Figure 3: Household Hours Worked and Female Labor-Force Participation, 1950-1990.

What economic factors can explain these facts? The idea here is that technological progress played a major role in inducing these changes.[5] Two hundred years ago the U.S. was largely a rural economy. The household was the basic production unit, with the family producing a large fraction of what it consumed. At the time, most marriages were arranged by the parents of young adults. Key considerations were whether or not the potential groom would be a good provider, and the bride a good housekeeper.[6] Over time more and more household goods and services could be purchased outside the home, such as packaged foods and ready-made clothes. Additionally capital goods, ranging from washing machines to microwave ovens, were brought into the home greatly reducing the time needed to maintain a household. This had two effects. First, it allowed all adults, both married and single, to devote more time to market activities and less to household production. Second, it lowered the economic incentives to get married by reducing the benefits of the traditional specialization of women at housework, and of men at market work. The reduction of the economic benefits of marriage allowed the modern criteria of mutual attraction between mates to come to the fore, a trend "from economics to romance" in the words of Ogburn and Nimkoff (1955).

To model this idea formally, a Becker (1965) - cum - Reid (1934) model of household production is embedded into a Mortensen (1988) style spouse-search model. There are two reasons for marriage in the framework: loosely speaking, love and money.[7] The economic reasons derive from the fact that when there are economies of scale in household consumption

[5] The impact of technological progress on household formation was addressed some time ago in a classic and prescient book by Ogburn and Nimkoff (1955). The book analyzes the impact of technological progress on family size, marriage and divorce, and female labor-force participation, among other things.

[6] Ogburn and Nimkoff (1955, pp. 40-41) quote Godey's *The Lady's Book* in 1831 as writing "No sensible man ever thought a beautiful wife was worth as much as one that could make good pudding" or in 1832 as stating "Among our industrious fore-fathers it was a fixed maxim that a young lady should never be permitted to marry until she had spun for herself a set of body, bed and table linen. From this custom all unmarried women are called spinsters in legal proceedings."

[7] The interaction between monetary and non-monetary incentives to get married is also analyzed by Fernandez, Guner and Knowles (2001). In their framework a higher level of inequality generates a higher degree of marital sorting. This occurs because the economic costs of marrying down increase for the rich. They also present empirical evidence supporting this prediction.

and production it pays for a couple to pool their resources together. Suppose that purchased household inputs and labor are substitutes in household production. Then, a fall in the price of purchased household inputs will displace labor from the home. Furthermore, if there is stronger diminishing marginal utility in nonmarket goods vis à vis market goods then married households will allocate a smaller fraction of their spending on the inputs for household production than will single households. As a consequence, single households gain the most from a decline in the price of purchased household inputs. Thus, a fall in the price of purchased household inputs causes the relative benefits of single life to increase. Singles searching for a spouse will become pickier. For those currently married, the value of a divorce will rise, because the value of becoming single is higher.

2 The Economic Environment

The economy is populated by a continuum of people with unit mass. Individuals have finite lives. Specifically, at the beginning of each period an individual faces the constant probability of dying, δ. Thus, δ people die each period. The individuals who have passed on are replaced by a newly-borne generation of exactly the same size. There are two types of individuals: those who are single and those who are married. Each individual is endowed with one unit of time, which can be divided between market and nonmarket work. A unit of market work pays the wage rate, w. At the beginning of each period singles participate in a marriage market, assuming that they have survived. Each single is randomly paired up with another one. The match will have a certain level of suitability or quality, b. The question facing a single is: should s/he marry, or wait until a better match comes along. For a married couple match quality evolves over time. For simplicity, assume that married couples die together at the start of a period with probability δ. If they survive then they must decide whether or not to remain married. After the marriage and divorce decisions, individuals enter the labor market. A single agent must decide how much of his one unit of time to devote to market work. A married couple must determine how much of their two units of time to spend in the labor force.

2.1 Tastes

Singles: Let the momentary utility function for a single read

$$U^s(c,n) = \alpha \ln(c - \mathfrak{c}) + (1-\alpha)n^\zeta/\zeta, \text{ with } \zeta < 0 < \alpha < 1.$$

Here c and n denote the person's consumption of market and nonmarket goods, respectively. The constant $\mathfrak{c}$ represents a fixed cost associated with maintaining a household.[8] This represents the first of two sources of scale economies in household consumption. If a single dies he realizes a utility level of zero in the afterlife, an innocuous normalization.

Married Individuals: Tastes for a married individual are given by

$$U^m(c,n) + b = \alpha \ln((c - \mathfrak{c})/2^\phi) + (1-\alpha)(n/2^\phi)^\zeta/\zeta + b, \text{ with } \zeta < 0 < \phi < 1,$$

where c and n represent the *household's* consumption of market and nonmarket goods. To determine an individual's consumption, $c - \mathfrak{c}$ and n are divided by the household equivalence scale, 2^ϕ, to get consumptions per member, $(c - \mathfrak{c})/2^\phi$ and $n/2^\phi$. Since $0 < \phi < 1$, this implies that it is less expensive to provide the second member of the household with consumption than it is the first. This is the second source of economies of scale in consumption. Note that the utility function for nonmarket goods is more concave than the ln function; i.e., $\zeta < 0$. Observe that the level of marital bliss from a match of quality, b, may be negative. Finally, if a married couple die they realize a zero-utility level thereafter.

2.2 Household Production

Suppose that nonmarket goods, n, are made in line with the following household production function:

$$n = [\theta d^\kappa + (1-\theta)h^\kappa]^{1/\kappa}, \text{ for } 0 < \kappa < 1, \tag{1}$$

where d denotes the household's purchases of household inputs, and h is the amount of time spent on housework.[9] Let purchased household inputs sell at price p, measured in terms

[8] See footnote 11 for more detail.

[9] For some uses of household production theory in macroeconomics see Benhabib, Rogerson and Wright (1991), Gomme, Kydland and Rupert (2001), Greenwood and Hercowitz (1991), and Parente, Rogerson and

of time. The idea here is that over time p will drop. Specifically, let p fall monotonically to some lower bound $\underline{p}> 0$. In response households will substitute out of using labor toward using more purchased inputs. Note that it has been assumed that purchased inputs and time are more substitutable in production than Cobb-Douglas; i.e., $\kappa > 0$. Hence, as p declines, household production will become more goods intensive and less labor intensive.[10]

2.3 Market Production

The production of market goods is done in line with the constant-returns-to-scale production technology

$$\mathbf{y} = w\mathbf{l}, \tag{2}$$

where $\mathbf{y}$ is aggregate output and $\mathbf{l}$ is aggregate employment. Given the linear form for the aggregate production function, w will represent the real wage rate in equilibrium. Real wages will grow over time. In particular, suppose that w increases monotonically to some finite upper bound $\overline{w}$. There is no financial or physical capital in the economy.

2.4 Match Quality

Recall that when singles meet they draw a match quality, b. Suppose that b is normally distributed so that

$$b \sim N(\mu_s, \sigma_s^2),$$

where μ_s and σ_s^2 are the mean and variance of the single distribution. Let the cumulative distribution function that singles draw from be represented by $S(b)$. Likewise, each period a married couple draws a new value for the match quality variable, b. Suppose that last period the couple had a match quality of b_{-1}. Now, assume that b evolves according to the following autoregressive process:

$$b = (1-\rho)\mu_m + \rho b_{-1} + \sigma_m\sqrt{1-\rho^2}\xi, \text{ with } \xi \sim N(0,1).$$

Wright (2000), and Rios-Rull (1993).

[10] Krusell, Ohanian, Rios-Rull and Violante (2000) employ this notion of labor-shedding technological progress in their study of the post-1974 rise in the skill premium.

Here μ_m and σ_m^2 denote the long-run mean and variance for the process b. The parameter ρ is the coefficient of autocorrelation. Write the (conditional) cumulative distribution function that married couples draw from as $M(b|b_{-1})$.

3 Household Decision Making

How will a single agent divide his or her time between market and nonmarket work? When will he or she choose to get married? Likewise, how will a married couple split their time between market work and housework? When will they choose to divorce? To answer these questions, let $V(b)$ denote the expected lifetime utility for an individual who is currently in a marriage with match quality b. Similarly, W will represent the expected lifetime utility for an agent who is single today. Imagine that two singles meet and draw a match quality of b. They will choose to marry if $V(b) \geq W$ and to remain single if $V(b) < W$. Likewise, consider a married couple with match quality b. They will pick to remain married when $V(b) \geq W$ and choose to divorce if $V(b) < W$. Thus, the marriage and divorce decisions are summarized by Table 1. So, how are the functions $V(b)$ and W determined? This question will be addressed next.

TABLE 1: MARRIAGE AND DIVORCE DECISIONS

Single			*Married*		
Marry	*if*	$V(b) \geq W$	Remain Married	*if*	$V(b) \geq W$
Remain Single	*if*	$V(b) < W$	Divorce	*if*	$V(b) < W$

3.1 Singles

The dynamic programming problem for a single agent appears as

$$W = \max_{c,n,d,h} \{U^s(c,n) + \beta \int \max[V'(b'), W'] dS(b')\}, \qquad \text{(P1)}$$

subject to

$$c = w(1-h) - wpd, \qquad (3)$$

and (1).[11] The discount factor β reflects the probability of dying. That is, if $\widetilde{\beta}$ is the person's subjective discount factor then $\beta = (1-\delta)\widetilde{\beta}$. Recall that wages are rising over time and that prices are falling. Thus, W and V are functions of time. Given this, W' and V' denote the value functions for single and married lives that will obtain next period. Observe that while the individual is single today, the agent picks married or single life next period to maximize welfare, as the term $\max[V'(b'), W']$ in (P1) makes clear.

3.2 Couples

The dynamic programming problem for a married couple reads

$$V(b) = \max_{c,n,d,h} \{U^m(c,n) + b + \beta \int \max[V'(b'), W'] dM(b'|b)\}, \tag{P2}$$

subject to

$$c = w(2-h) - wpd, \tag{4}$$

and (1). Problem (P2) is similar in structure to problem (P1) with three differences: (i) the utility function for married agents differs from single agents due to scale effects in household consumption, (ii) a married couple realizes bliss from marriage and this is autocorrelated over time, and (iii) the couple has two units of time to allocate between market and nonmarket work. Again, note that while an individual is married today, the agent chooses married or single life next period to maximize welfare.[12]

[11] It can now be seen that the constant $\mathfrak{c}$ does represent a fixed cost associated with maintaining a household. Write U^s as $U^s(\widetilde{c}, n)$, where $\widetilde{c}$ denotes the consumption of market goods by a single household. Let $\mathfrak{c}$ denote the fixed cost of maintaining a household. The household's budget constraint will now appear as $\widetilde{c} = w(1-h) - wpd - \mathfrak{c}$. Rewrite the budget constraint as $\widetilde{c} + \mathfrak{c} = w(1-h) - wpd$. Next, define $c = \widetilde{c} + \mathfrak{c}$ so that $\widetilde{c} = c - \mathfrak{c}$. Use this to substitute out for $\widetilde{c}$ in U^s and the above budget constraint. This setting has transformed into the one presented in the text.

[12] The structure of problems (P1) and (P2) is similar to the typical search/work problem – see Hansen and Imrohoroglu (1992), Jovanovic (1987), and Wright and Loberg (1987) for some examples, or Rogerson, Shimer and Wright (2004) for a recent survey.

4 Equilibrium

Formulating an equilibrium to the above economy is surprisingly simple. First, given the linear market production function (2), there is no need to determine the equilibrium wage, w. Second, since there are no financial markets, there is no interaction between households other than through the marriage market. As far as consumption and production are concerned, each household is more or less an island unto themselves. Hence, characterizing an equilibrium for the economy amounts to solving the programming problems (P1) and (P2). Thus, it is easy to establish that an equilibrium for the above economy both exists and is unique.[13]

4.1 Vital Statistics

Computing vital statistics for the economy is a relatively straightforward task. Suppose that the economy exits the previous period with the (non-normalized) distribution $\mathbf{M}_{-1}(b_{-1})$ over match quality for married agents. The fractions of agents who were married and single last period, m_{-1} and s_{-1}, are therefore given by $m_{-1} = \int d\mathbf{M}_{-1}(b_{-1})$ and $s_{-1} = 1 - \int d\mathbf{M}_{-1}(b_{-1})$. Now, at the beginning of the current period the fraction δ of populace dies. These people are replaced by newly-borne single agents. All agents will then take a draw, b, for their match quality. After this, they will make their marriage and divorce decisions in line with Table 1. Define the set of match quality shocks for which it is in an individual's best interest to live in a married household, or $\mathcal{M}$, by

$$\mathcal{M} = \{b : V(b) \geq W\}.$$

[13] In a nutshell the argument is as follows: Recall that $\lim_{t\to\infty} p_t = \underline{p}$ and $\lim_{t\to\infty} w_t = \overline{w}$. It is easy to deduce that a pair of unique steady-state value functions, W^* and V^*, exist that will solve the dynamic programming problems (P1) and (P2). To see this note that (P1) and (P2) define an operator $(W, V) = \mathbf{T}(W', V')$. By standard arguments, it can be readily deduced that the operator $\mathbf{T}$ is a contraction mapping on the space of continuous bounded functions with norm $||W, V|| = \sup|W| + \sup|V|$. Working backwards in time from the steady state it is also easy to see that the value functions W and V will exist and be unique at each stage of the recursion.

The current-period distribution over match quality for married agents, or $\mathbf{M}(b)$, will then read[14]

$$\mathbf{M}(b) = (1-\delta)\int_{\mathcal{M}\cap[0,b]}\int dM(\widetilde{b}|b_{-1})d\mathbf{M}_{-1}(b_{-1}) + [s_{-1}+\delta m_{-1}]\int_{\mathcal{M}\cap[0,b]} dS(\widetilde{b}).$$

Therefore, the fractions of agents who are married and single in current period, m and s, are given by $m = \int d\mathbf{M}(b)$ and $s = 1 - \int d\mathbf{M}(b)$. The fraction of people getting married in the current period is $[s_{-1}+\delta m_{-1}]\int_{\mathcal{M}} dS(\widetilde{b})$, while the proportion going through a divorce is given by $(1-\delta)\int_{\mathcal{M}^c}\int dM(\widetilde{b}|b_{-1})d\mathbf{M}_{-1}(b_{-1})$, where $\mathcal{M}^c$ is the complement of $\mathcal{M}$.

5 Theory

It is now time to entertain the following two questions, at least at a theoretical level:

1. How does technological progress affect amount of time spent on housework?

2. How does technological progress affect the economic return from married versus single life?

Before answering these two questions, it may pay to take stock of the key features of the model and to discuss the role that they play in the subsequent analysis.

- *Household Equivalence Scale*, $0 < \phi < 1$. This provides an economic incentive for marriage. If a two-person household can live more economically than a single-person household then there are gains from marriage.

- *Purchased Household Inputs-Housework Substitutability in Home Production*, $0 < \kappa < 1$. Suppose that over time the price of purchased household inputs declines. The higher the degree of substitutability between purchased inputs and housework the bigger will be the labor-saving impact of technological progress on home production.

[14] Note that when a single agent dies he is replaced by another single agent. This explains why there is no term reflecting the probability of dying multiplying s_{-1} in the formula for $\mathbf{M}(b)$.

- *Strong Diminishing Marginal Utility for Nonmarket Goods*, $\zeta < 0$. Married couples will consume more of all goods than singles do. If the utility function for nonmarket goods is more concave than the one for market goods then married couples will allocate a higher fraction of their spending (as compared with singles) to market goods, c, relative to inputs into home production, d and h.

- *Fixed Cost of Household Maintenance*, $\mathfrak{c} > 0$. This gives another economic incentive for marriage. Moreover, at low levels of income households will have limited resources to allocate for market consumption, after meeting the fixed cost of household maintenance. This will force poorer households, or singles, to devote a higher fraction of their time to market work relative to richer households, or married couples.

- *Marital Bliss*, b. This creates a noneconomic incentive for marriage.

- *The Probability of Dying*, $0 < \delta < 1$. This proves useful in the quantitative analysis. It increases the fraction of people who are single, ceteris paribus.

The task is now to establish that the features outlined above do indeed play their assigned roles.

5.1 The Time-Allocation Problem

The Problem: To this end, consider the time-allocation problem that faces a household of size, z. It is static in nature and appears as

$$I(z,p,w) = \max_{c,n,h,d}\{\alpha \ln(\frac{c-\mathfrak{c}}{z^{\phi}}) + (1-\alpha)(\frac{n}{z^{\phi}})^{\zeta}/\zeta\}, \tag{P3}$$

subject to

$$c - \mathfrak{c} = w(z-h) - wpd - \mathfrak{c}, \tag{5}$$

and

$$n = [\theta d^{\kappa} + (1-\theta)h^{\kappa}]^{1/\kappa}.$$

Observe that versions of problem (P3) are embedded into (P1) and (P2), a fact that can be seen by setting $z = 1$ and $z = 2$.

The Solution: By using the constraints for n and $c - \mathfrak{c}$ in the objective function (P3), and then maximizing with respect to d and h, the following two first-order conditions are obtained:

$$\frac{\alpha}{c - \mathfrak{c}} wp = (1 - \alpha) z^{-\phi\zeta} [\theta d^{\kappa} + (1 - \theta) h^{\kappa}]^{\zeta/\kappa - 1} \theta d^{\kappa - 1}, \tag{6}$$

and

$$\frac{\alpha w}{c - \mathfrak{c}} = (1 - \alpha) z^{-\phi\zeta} [\theta d^{\kappa} + (1 - \theta) h^{\kappa}]^{\zeta/\kappa - 1} (1 - \theta) h^{\kappa - 1}. \tag{7}$$

These two first-order conditions have standard interpretations. For instance, the left-hand side of (6) represents the marginal cost of an extra unit of purchased household inputs, d. The marginal unit of purchased household inputs costs wp in terms of forgone market consumption. Since an extra unit of market consumption has a utility value of $\alpha/(c - \mathfrak{c})$ this leads to a sacrifice of $[\alpha/(c - \mathfrak{c})]wp$ in terms of forgone utility. Likewise, the right-hand side of this equation gives the marginal benefit of an extra unit of purchased household inputs. These extra goods will increase household production by $[\theta d^{\kappa} + (1 - \theta) h^{\kappa}]^{1/\kappa - 1} \theta d^{\kappa - 1}$. The marginal utility of nonmarket goods is $(1 - \alpha) z^{-\phi\zeta} (n)^{\zeta - 1}$. Thus, the marginal benefit of an extra unit of purchased household inputs is $(1 - \alpha) z^{-\phi\zeta} (n)^{\zeta - 1} \times [\theta d^{\kappa} + (1 - \theta) h^{\kappa}]^{1/\kappa - 1} \theta d^{\kappa - 1}$, which is the right-hand side of (6).

Next, combining (6) and (7) yields

$$d = [\frac{(1 - \theta) p}{\theta}]^{1/(\kappa - 1)} h \equiv R(p) h. \tag{8}$$

Using this in (5) then gives

$$c - \mathfrak{c} = w[(z - \frac{\mathfrak{c}}{w}) - h] - wpd = w[(z - \frac{\mathfrak{c}}{w}) - h] - wpR(p)h. \tag{9}$$

Finally, by substituting (8) and (9) into (7) a single equation can be obtained in one unknown, namely h:

$$\alpha[\theta R(p)^{\kappa} + (1 - \theta)]^{1 - \zeta/\kappa} h^{1 - \zeta} = (1 - \alpha)(1 - \theta) z^{-\phi\zeta} [(z - \frac{\mathfrak{c}}{w}) - h - pR(p)h]. \tag{10}$$

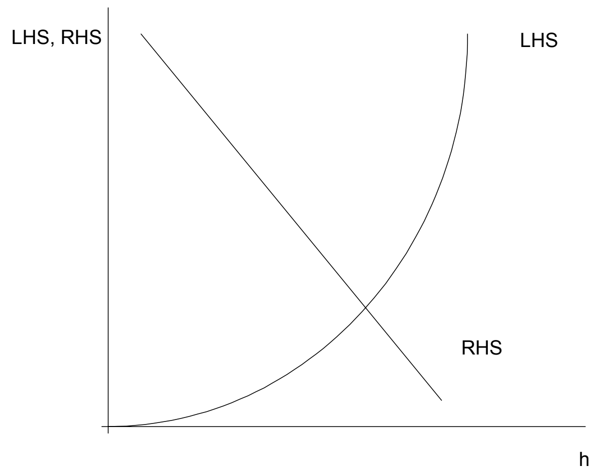

Figure 4: The Determination of h.

The solution is portrayed in Figure 4. It is easy to deduce that the left-hand side of (10) is increasing in h, since $\zeta < 0$. It is trivial to see that the right-hand side is decreasing in h.

5.2 Results

Everything is now set up to address the two questions poised at the start of this section.

Technological Progress and Time Allocations: So, how does technological progress affect the amount of time allocated to homework? First, a fall in the price of purchased household inputs, p, leads to a reduction in the amount of housework, h, and a rise in the amount of market work, $z - h$. When the price of purchased household inputs drops households substitute away from using labor in household production toward using goods. Second, a rise in wages, w, leads to an increase in the amount of housework, h, done. At low levels of income, the marginal utility of market goods is high due to the fixed cost of household maintenance, $\mathfrak{c}$. Thus, people devote a lot of time to laboring in the market. As wages increase the fixed cost for household maintenance bites less and people relax their work effort in market.

Proposition 1 *Housework, h, is:*
(i) increasing in the price of household commodities, p;
(ii) increasing in real wages, w.

Proof. (i) Observe that both $R(p) = \{[(1-\theta)/\theta]p\}^{1/(\kappa-1)}$ and $pR(p)$ are decreasing in p, since $0 < \kappa < 1$. Therefore, the right-hand side of (10) falls with a drop in p, as $-pR(p)$ is increasing in p. Thus, the RHS curve in Figure 4 will shift down when p declines. The left-hand side increases with a reduction in p because $R(p)^{\kappa}$ is decreasing in p. Hence, the LHS curve shifts up. As a consequence, h unambiguously drops. (ii) It's trivial to see that the right-hand side of (10) is increasing in w, while the left-hand side is not a function of w. Therefore, an increase in w will cause h to rise. ■

Corollary *Housework, h, is decreasing in the fixed cost of household maintenance, $\mathfrak{c}$.*

Proof. Note that w and $\mathfrak{c}$ only enter into (10) in the form $\mathfrak{c}/w$. ■

Remark *Observe that wages, w, will have no effect on time allocations in the absence of a fixed cost for household maintenance*, $\mathfrak{c}$; *i.e., when $\mathfrak{c} = 0$. Thus, as the economy develops the impact of wages on housework will vanish, since $\mathfrak{c}/w \longrightarrow 0$ as $w \longrightarrow \infty$.*

Household Size and Allocations: What is the relationship between the size of a household, on the one hand, and the amount of time allocated to housework and spending on goods, on other hand? One would expect housework, h, to rise when size, z, increases because the total endowment of time has risen. This is true. A more interesting question is whether or not housework rises by a factor more or less than the proportionate increase in household size. On the one hand, given that the utility function for nonmarket goods is more concave than the one for market goods, the household has a preference for diverting extra resources into market consumption. This suggests that housework will increase less than proportionately with size. On the other hand, at higher levels of income the fixed cost for household maintenance will matter less. This propones that housework will rise more than proportionately with size. While the result turns out to be ambiguous, a useful upper bound on the response of housework to household size can be derived. Using this upper bound, it can be shown that married households spend less than single households do on the inputs into household production, d and h, at least relative to market consumption, $c - \mathfrak{c}$.

Lemma 2 *A rise in z by a factor of $\lambda > 1$ leads to an increase in h by a factor strictly less than $\rho = (\lambda z - \mathfrak{c}/w)/(z - \mathfrak{c}/w)$. When $\zeta = 0$ (*ln *utility for nonmarket goods) a magnification in z by a factor of $\lambda > 1$ will cause h to expand by exactly a factor of ρ.*

Proof. Rewrite equation (10) as

$$\alpha[\theta R(p)^{\kappa} + (1-\theta)]^{1-\zeta/\kappa} h^{-\zeta} h + (1-\alpha)(1-\theta)(1+pR(p)) z^{-\phi\zeta} h$$
$$= (1-\alpha)(1-\theta) z^{-\phi\zeta}(z - \mathfrak{c}/w).$$

If z increases by factor $\lambda > 1$ then $z - \mathfrak{c}/w$ rises by the factor $\rho \equiv (\lambda z - \mathfrak{c}/w)/(z - \mathfrak{c}/w) > \lambda$. Now, the right-hand side rises by the factor $\lambda^{-\phi\zeta}\rho$. Observe that if h rises by the factor ρ then the left-hand side will increase by more than the factor $\lambda^{-\phi\zeta}\rho$, because $\rho^{-\zeta} > \lambda^{-\phi\zeta}$ when $\zeta < 0$ and $0 < \phi < 1$. Therefore, to restore equality between the left-hand and right-hand sides of the above equation, h must rise by less than the factor ρ. The first part of the lemma has been established. Last, suppose that $\zeta = 0$. In this case, (10) reduces to

$$h = \frac{(1-\alpha)(1-\theta)(z - \mathfrak{c}/w)}{\alpha[\theta R(p)^{\kappa} + (1-\theta)] + (1-\alpha)(1-\theta)(1+pR(p))}. \tag{11}$$

The second part of the lemma follows immediately. ■

Can anything be said about the allocations, (c, d, h), within a two-person household vis à vis a one-person household? The lemma below provides the answer, where the superscripts m and s are attached to the allocations for married and single households. Before proceeding, it will be noted that the lemma is a key step along the road to proving that a fall in the price for purchased household inputs reduces the utility differential between married and single life, when holding fixed the amount of marital bliss. It shows that a married household spends less on purchased household inputs, relative to market consumption (over and above the fixed cost of household maintenance), than does a single one. Likewise, the corollary to the lemma is instrumental for establishing that a rise in wages reduces the economic benefit from marriage. It proves that a married household consumes more market goods than a single household does.

Lemma 3 *The allocations in married and single households have the following relationships:*
(i) $(c^m - \mathfrak{c}) > [(2 - \mathfrak{c}/w)/(1 - \mathfrak{c}/w)](c^s - \mathfrak{c})$;
(ii) $d^m < [(2 - \mathfrak{c}/w)/(1 - \mathfrak{c}/w)]d^s$;
(iii) $h^m < [(2 - \mathfrak{c}/w)/(1 - \mathfrak{c}/w)]h^s$.
The above relationships hold with equality when $\zeta = 0$.

Proof. First, result (iii) is immediate from Lemma (2). Second, it is easy to see that (ii) is implied by equation (8) and result (iii). By using (ii) and (iii), in conjunction with equation (9), result (i) can be obtained. Last, the situation for $\zeta = 0$ is readily handled by using the closed-form solution (11). ■

In line with the intuition presented above on the relationship between household size and allocations, suppose that $\mathfrak{c} = 0$. In this case, $\rho = \lambda$. Thus, larger households will devote proportionately less of their time to housework than smaller ones, since an increase in household size by a factor $\lambda > 1$ will lead to a rise in h by a factor less than $\rho = \lambda$. Note that conditions (i) to (iii) in Lemma 3 still hold with strict inequality in this case so that the results do not depend on the presence of the fixed cost. Next, suppose that $\zeta = 0$ and $\mathfrak{c} > 0$, so that both market goods and nonmarket goods have ln utility. Conditions (i) to (iii) now hold with equality. If z increases by a factor of λ then h will rise by exactly the factor $(\lambda z - \mathfrak{c}/w)/(z - \mathfrak{c}/w) > \lambda$. Now, larger households spend proportionately more of their time on housework relative to smaller ones. Finally, the fact that the above conditions hold with equality when $\zeta = 0$ and with strict inequality when $\zeta < 0$ (regardless of whether $\mathfrak{c} = 0$ or $\mathfrak{c} > 0$), implies that the assumption of strong diminishing marginal utility for nonmarket goods ($\zeta < 0$) is important for analyzing the impact of a drop in the price of purchased household inputs on the economic return to marriage, while the presence of a fixed cost of household maintenance ($\mathfrak{c} > 0$) is not – this statement follows from the first part of the proof of Proposition 4.

Corollary *Married households consume more market goods than do single households:*
(i) $(c^m - \mathfrak{c})/2^{\phi} > (c^s - \mathfrak{c})$;

(ii) $c^m > c^s$.

Proof. From Lemma 3, note that when $\zeta \leq 0$ it transpires that $(c^m - \mathfrak{c}) \geq 2(c^s - \mathfrak{c})$, since $[(2 - \mathfrak{c}/w)/(1 - \mathfrak{c}/w)] \geq 2$. Thus, $(c^m - \mathfrak{c})/2^\phi \geq 2^{1-\phi}(c^s - \mathfrak{c}) > (c^s - \mathfrak{c})$. Part (ii) of the corollary follows trivially. ∎

Now, note that a married household has $2 - \mathfrak{c}/w$ units of disposable time, after netting out the fixed cost of household maintenance, to spend on various things. A single household has $1 - \mathfrak{c}/w$ units of disposable time. Lemma 3 states that a married household will spend a larger fraction of their time endowment on the consumption of market goods than will a single household. The lemma also implies that married households spend less than single households do on household inputs, relative to market goods. That is, $pd^m/(c^m - \mathfrak{c}) < pd^s/(c^s - \mathfrak{c})$ and $wh^m/(c^m - \mathfrak{c}) < wh^s/(c^s - \mathfrak{c})$ so that $[pd^m + wh^m]/(c^m - \mathfrak{c}) < [pd^s + wh^s]/(c^s - \mathfrak{c})$. Part (i) of the corollary states that after paying the fixed cost of household maintenance, market consumption per person is effectively higher in a married household than a single one. Also, married households spend more in total on market goods than do single households. The corollary is true even when $\zeta = 0$. As can be easily seen, its proof requires that the inequalities in the Lemma 3 hold only *weakly*. This implies that in the subsequent analysis the assumption of strong diminishing marginal utility for nonmarket goods is *not* important for proving that a rise in wages reduces the economic return to marriage – this statement follows from the second part of the proof of Proposition 4. All that is required is the presence of a fixed cost for household maintenance.

Technological Progress and the Economic Benefits of Married versus Single Life: Last, how does technological progress affect the utility differential between married and single life (holding fixed the amount of marital bliss)? To address this, let u^m denote the level of momentary utility realized from married life, sans marital bliss, and u^s represent the level of utility realized from single life. From problem (P3) it is apparent that $u^m = I(2, p, w)$ and $u^s = I(1, p, w)$.

Proposition 4 *The utility differential between married and single life (sans marital bliss),* $u^m - u^s$*, is:*

(i) increasing in the price of purchased household inputs, p;
(ii) decreasing in real wages, w.

Proof. The first part of the lemma can be established by applying the envelope theorem to problem (P3). It can be calculated that

$$\frac{d(u^m - u^s)}{dp} = -\alpha w[\frac{d^m}{c^m - \mathfrak{c}} - \frac{d^s}{c^s - \mathfrak{c}}] > 0, \tag{12}$$

where the sign of the above expression follows from Lemma 3(i, ii). To prove the second part of the lemma, note that

$$\begin{aligned}\frac{d(u^m - u^s)}{dw} &= \alpha[\frac{2 - h^m - pd^m}{c^m - \mathfrak{c}} - \frac{1 - h^s - pd^s}{c^s - \mathfrak{c}}] \\ &= \frac{\alpha}{w}[\frac{c^m}{c^m - \mathfrak{c}} - \frac{c^s}{c^s - \mathfrak{c}}] = \frac{\alpha}{w}[\frac{1}{1 - \mathfrak{c}/c^m} - \frac{1}{1 - \mathfrak{c}/c^s}] < 0,\end{aligned}$$

where the sign of the above expression derives from the fact that $c^m > c^s$, or part (ii) of the corollary to Lemma 3. ■

Thus, technological advance in the form of either a falling price for purchased household inputs or rising real wages reduces the economic gain from marriage. A fall in the price of purchased household inputs leads to a substitution away from the use of labor in household production toward the use of purchased household inputs. Single households use labor-saving products the most intensively [i.e., $d^m/(c^m - \mathfrak{c}) < d^s/(c^s - \mathfrak{c})$], so they realize the greatest gain.[15] As wages increase, the fixed cost for household maintenance matters less. The fixed cost for household maintenance bites the most for single households [i.e., $\mathfrak{c}/c^m < \mathfrak{c}/c^s$]. Therefore, single households benefit more from a rise in wages.

What is the monetary value of married life? One way to measure this is to compute the required income, or compensation, that is necessary to make a single person as well off as

[15] While Proposition 4 is very suggestive, given the general nature of adopted setup it is hard to say much concrete about the impact of technological progress on marriage and divorce, per se. To do so requires either restricting the theoretical setup or numerically simulating the model. The latter is done in the next section. The former strategy is pursued in the Appendix. It can be shown that a one-period decline in the price for purchased household inputs leads to a one-period drop in the rate of marriage and a one-period increase in the divorce rate. By specializing the stochastic structure of the model further, it can be established that a steady-state decline in the price for purchased household inputs leads to fall in the steady-state fraction of the population that is married.

a married one with no bliss ($b = 0$). This can be done by solving the following expenditure problem:

$$E(p, w, u^m) = \min_{\widetilde{c}^s, \widetilde{d}^s, \widetilde{h}^s} \{\widetilde{c}^s + pw\widetilde{d}^s + w\widetilde{h}^s\}, \tag{P4}$$

subject to (1) and

$$\mu \ln(\widetilde{c}^s - \mathfrak{c}) + (1 - \mu)(\widetilde{n}^s)^\zeta/\zeta = u^m. \tag{13}$$

Equation (13) states that the momentary utility level realized by a single agent must equal that of a married one with no bliss, or u^m. Hence, problem (P4) finds the minimum level of expenditure that makes a single person as well off as a married one. A consumption-based measure of the economic benefits from marriage is then given by

$$\ln[\frac{E(p, w, u^m)}{w}],$$

where w is the value of a single agent's time endowment. The solution to the expenditure problem is surprisingly simple and natural.

Lemma 5 *The compensating differential between married and single life is given by*

$$\ln[E(p, w, u^m)/w] = \ln[2^{1-\phi} + (1 - 1/2^\phi)\mathfrak{c}/w].$$

Proof. The solution to expenditure problem (P4) is once again characterized by (7) and (8) for $z = 1$, in conjunction with (13). Now, suppose that $c^m - \mathfrak{c}$, d^m, and h^m satisfy the married time-allocation problem or (P3) when $z = 2$. Then, it is easy to show that $\widetilde{c}^s - \mathfrak{c} = (c^m - \mathfrak{c})/2^\phi$, $\widetilde{d}^s = d^m/2^\phi$, and $\widetilde{h}^s = h^m/2^\phi$ satisfy the expenditure problem. Given this, it follows that $\widetilde{c}^s + pw\widetilde{d}^s + w\widetilde{h}^s = [(c^m - \mathfrak{c}) + pwd^m + wh^m]/2^\phi + \mathfrak{c} = w/2^\phi + \mathfrak{c}$. The above result obtains. ■

The result is very appealing and the underlying intuition straightforward. For expositional purposes, let $\mathfrak{c} = 0$. On the one hand, a married household has twice the time endowment of a single one. On the other hand, a married household must provide consumption to twice as many members. On net, due to economies of scale in household consumption, a married household realizes $2^{1-\phi}$ ($=2/2^\phi$) as much consumption as a single one. Now, when

$\mathfrak{c} > 0$ an adjustment must be made for the presence of the fixed cost of household maintenance. This reduces a single's consumption by $\mathfrak{c}$ but a married's one by only $\mathfrak{c}/2^{\phi}$, so that the difference is $(1 - 1/2^{\phi})\mathfrak{c}$. Finally, note that the income needed to make a single person as well off as a married one is not a function of the price of purchased household inputs; one just needs to scale up a single's income by the constant fraction $2^{1-\phi} + (1 - 1/2^{\phi})\mathfrak{c}/w$. It is a function of the wage rate, though. At higher wages rates the fixed cost bites less.

It may seem a bit puzzling that a fall in price reduces the utility differential between married and single life, $u^m - u^s$, but has no impact on the compensating differential between these two situations, $\ln[2^{1-\phi} + (1 - 1/2^{\phi})\mathfrak{c}/w]$. Suppose that one makes the compensation outlined by (P4). Then, married and single households will use labor-saving products in the same intensity, in the sense that $d^m/(c^m - \mathfrak{c}) = d^s/(c^s - \mathfrak{c})$ – see the proof of Lemma 5. This implies that any further change in price will have no impact on the utility differential, $u^m - u^s$, as can readily be seen from (12). Thus, for price changes the compensation only has to be done once; that is, once the compensation has been made a subsequent price change affects married and single households commensurately. This suggests that for tracking over time the impact of technological progress on the utility differential from marriage the compensating differential is not a perfect measure.

6 Quantitative Analysis

The theoretical analysis suggests that framework developed has promise for explaining the observed rise in the number of single households, together with the increase in hours worked by married ones. To gauge the quantitative potential of the framework, the model must be solved numerically. At the outset it will be stated that the goal of the analysis is not to simulate an all-inclusive model of household formation and labor-force participation. Rather, the idea here is to see whether or not the simple mechanisms put forth have the potential quantitative power to explain the postwar observations on household formation and labor-force participation. This is done without regard to the many other possible explanations for the same set of facts – some of which could be embedded into a more general version of the developed framework. Theory, by its essence, is a process of abstraction. Thus, some factors that may be important for understanding the phenomena under study have been left out of the analysis, both for purposes of clarity and tractability.

The Household's Dynamic Programming Problems – A Restatement: Given the static nature of the household's time allocation problem (P3), note that the dynamic programming problems for single and married households (P1) and (P2) can be rewritten as

$$W = I(1, p, w) + \beta \int \max[V'(b'), W'] dS(b'),$$

and

$$V(b) = I(2, p, w) + b + \beta \int \max[V'(b'), W'] dM(b'|b).$$

Here $I(z, p, w)$ gives the maximal level of momentary utility that a z-person household can obtain, given that the price of purchased household inputs is p and that the wage rate is w. The fact that for a household of a particular size, z, it is possible to calculate their current level of utility, $I(z, p, w)$, without regard to their marriage/divorce decision is very useful. Given a sequence of prices and wages, $\{p_t, w_t\}_t^{\infty}$, it possible to compute from (P3) the associated sequence of momentary utilities for single and married households, $\{I(1, p_t, w_t), I(2, p_t, w_t)\}_t^{\infty}$.

6.1 Matching the Model with the Data

In order to simulate the model numbers must be assigned to the various parameters. Except for five of the parameters, almost nothing is known about appropriate values. Additionally, time series for prices and wages need to be inputted into the simulation. Take the model period to be one year. The parameter values used in the analysis are presented in Table 2. Their determination will now be discussed.

In line with convention, set the subjective discount factor at 0.96. The discount factor used in decision making must reflect the individual's probability of survival, $1-\delta$. A person's life expectancy is $1/\delta$. Thus, if (marriageable) life expectancy for an adult is taken to be 47 years then $1/\delta = 47$. Therefore, set $\beta = 0.096 \times (1 - 1/47)$. Next, let $\phi = 0.77$. This is in line with the O.E.C.D.'s household equivalence scale that treats the second adult in a family as consuming an additional 0.7 times the amount of the first adult. Hence, the parameter ϕ solves $1/2^{\phi} = 1/(1.0 + 0.7)$. A series for wages can be constructed from the U.S. data.[16] Between 1950 and 2000 compensation per hour worked rose 2.3 times. Thus, the analysis simply presumes that wages rise at $100 \times \ln(2.3)/50 = 1.7$ percent per year.

Table 2: Parameter Values

Tastes	$\beta = 0.960 \times (1-\delta)$, $\alpha = 0.278$, $\zeta = -1.750$, $\phi = 0.766$
Technology	$\mathfrak{c} = 0.119$, $\theta = 0.206$, $\kappa = 0.189$
Life span	$1/\delta = 47$
Shocks	$\mu_s = -4.219, \sigma_s^2 = 8.750$
	$\mu_m = 0.578$, $\sigma_m^2 = 0.568, \rho = 0.874$
Prices	$p_{1950} = 11.218$, $\gamma = 0.057$
	$p_t = p_{1950} \times e^{-\gamma \times (t-1950)}$ for $t = 1951, ..., 2000$
Wages	$w_{1950} = 1.00$
	$w_t = w_{1950} \times e^{0.017 \times (t-1950)}$ for $t = 1951, ..., 2000$

6.1.1 Household Technology Parameters

Obtaining a price series for purchased household inputs is somewhat problematic. So, a time path of the form $p_t = p_{1950} \times e^{-\gamma(t-1950)}$ will be estimated here, where γ is the rate of decline in the time-price for purchased household inputs. Thus, five household technology parameters need to be determined, viz $\mathfrak{c}, \kappa, \theta, p_{1950}$, and γ. Three parameters are crucial for determining the pattern of time allocations, $\mathfrak{c}$, κ, and γ. The fixed cost for household maintenance, $\mathfrak{c}$, plays an important role in controlling the initial level of market work expended by singles relative to married households. Nothing is known about its value, so it also will be estimated. The elasticity of substitution between capital and labor in household production, $1/(1-\kappa)$, governs the responsiveness of housework to changes in the price of purchased household

[16] This is done by taking the series for GDP from the National Income and Product Accounts and dividing it through by Hours Worked by Full-Time and Part-Time Employees, both taken from the Bureau of Economic Analysis, U.S. Department of Commerce.

inputs. To match a given rise in the fraction of household time spent on market work, a low elasticity of substitution between household inputs can be partially compensated for by picking a high price decline, or vice versa. Values for κ and θ have been estimated by McGrattan, Rogerson and Wright (1997). Their numbers are used here.

To match the model up with the data on time allocations, note that the fraction of time spent by a married household on market work, l^m, is given by $l^m = (2 - h^m)/2$. Likewise, the fraction of time spent by a single household working in the market is $l^s = 1 - h^s$. Now, note that l^m and l^s can be written as functions of the parameters to be estimated, $\mathfrak{c}$, p_{1950}, and γ. They are also functions of time, t, and the taste parameters α and ζ. For the moment, assume that these two taste parameters have been determined somehow. Thus, write $l^m = L^m(\mathfrak{c}, p_{1950}, \gamma; t, \alpha, \zeta)$ and $l^s = L^s(\mathfrak{c}, p_{1950}, \gamma; t, \alpha, \zeta)$.

To calculate the analogous numbers for the U.S. data, assume that there are 112 nonsleeping hours in a week. Following the footsteps of McGrattan and Rogerson (1998), weekly hours per married and single households can be calculated using U.S. Census Data. For each decennial year between 1950 and 1990 the Census provides hours per week in following intervals: 1-14, 15-29, 30-34, 35-39, 40, 41-48, 49-59, and more than 60 hours. Let E_i denote the number of people that report hours in a particular interval i, E_R represent the total number of people reporting hours, E stand for the total number of people employed, and N be the total population. Then, the fraction of total nonsleeping time allocated to the market is calculated as

$$(7.5E_{1-14}+22E_{15-29}+32E_{30-34}+37E_{35-39}+40E_{40}+44.5E_{41-48}+42E_{49-59}+62.5E_{60+})\frac{1}{E_R}\frac{E}{N}\frac{1}{112}.$$

This fraction is computed by marital status for all males and females between ages 24 and 54. The fractions of total household time allocated to the market by married households, $\widetilde{l}^m$, and by single households, $\widetilde{l}^s$, are then calculated as the averages across male and female hours. Thus, an observation for $\widetilde{l}_t^m$ and $\widetilde{l}_t^s$ is obtained for each decade t between 1950 and 1990, inclusive.

The estimation procedure is thus described by

$$H(\alpha,\zeta)=\min_{\mathfrak{c},p_{1950},\gamma}\{\sum_{t\in\mathcal{T}}\omega_t[\widetilde{l}_t^m-L^m(\mathfrak{c},p_{1950},\gamma;t,\alpha,\zeta)]^2+(1-\omega_t)[\widetilde{l}_t^s-L^s(\mathfrak{c},p_{1950},\gamma;t,\alpha,\zeta)]^2/5\}, \tag{P5}$$

where $T=\{1950,1960,\cdots,1990\}$.[17] The above estimation scheme weights the time-t allocations of married and single households by ω_t and $1-\omega_t$. Here ω_t is defined to be the fraction of married females in the time-t population of women. The theory developed suggests that the parameters $\mathfrak{c}$, p_{1950}, and γ will be important for determining the time paths for hours worked. This suggests the time paths for hours worked may contain valuable information for determining the magnitudes of $\mathfrak{c}$, p_{1950}, and γ that should be exploited in the estimation procedure. The upshot of the estimation procedure will be discussed in Section 6.2, but it is interesting to note that it selects $\gamma=-0.057$. Thus, the time-price for purchased household inputs falls at 5.7 percent per year. This looks reasonable.[18] Last, note that the minimized value of the objective function in (P5) will depend upon the parameters α and ζ.

6.1.2 Taste and Matching Parameters

Seven taste and matching parameters remain to be discussed; namely, α, ζ, μ_s, σ_s, μ_m, σ_m, and ρ. Parameter α determines the weight of market goods in the utility function, while the parameter ζ controls the degree of concavity in the utility function for nonmarket goods. The more concave this utility function is the faster households will move away from nonmarket goods toward market goods as income rises. Hence, this parameter plays an important role in determining how the relative benefits of married versus single life respond to technological progress. The idea here is that information on the trend in vital statistics is important for the determining the value of ζ. The remaining five matching parameters govern the noneconomic aspects of marriage.

[17] The estimation procedure employed is similar to one used by Andolfatto and MacDonald (1998). Note that census data has 10 year periodicity. Thus, there are only 5 years of data. Given the paucity of observations there is little point in adding an error structure to the estimation.

[18] For instance, the Gordon quality-adjusted time price index for airconditioners, clothes dryers, dishwashers, microwaves, refrigerators, TVs, VCRs, and washing machines fell at 10 percent a year over the postwar period.

These seven parameters are chosen so that model matches up with, as well as possible, the U.S. vital statistics on marriage and divorce. In particular, they are picked so that the initial and final steady states of the model economy are close to the data for the years 1950 and 2000, respectively. The data is targeted along three dimensions: the fraction of population married, the divorce rate, and the marriage rate. This matching procedure is done along the lines of problem (P5). Specifically, let $m^i_{1950} = M^i(\alpha, \zeta, \mu_s, \sigma_s, \mu_m, \sigma_m, \rho; 1950)$ and $m^i_{2000} = M^i(\alpha, \zeta, \mu_s, \sigma_s, \mu_m, \sigma_m, \rho; 2000)$ represent the model's steady-state output along the i-th dimension for the years 1950 and 2000 (for $i = 1, 2, 3$). This output is a function of the parameters to be estimated. The matching procedure is then summarized by the minimization problem

$$\min_{\alpha\in\mathcal{A},\zeta,\mu_s,\sigma_s,\mu_m,\sigma_m,\rho} \{\frac{3}{5}\{\sum_{i=1}^{3}(1/3)[\widetilde{m}^i_{1950} - M^i(\alpha, \zeta, \mu_s, \sigma_s, \mu_m, \sigma_m, \rho; 1950)]^2$$

$$+ \sum_{i=1}^{3}(1/3)[\widetilde{m}^i_{2000} - M^i(\alpha, \zeta, \mu_s, \sigma_s, \mu_m, \sigma_m, \rho; 2000)]^2\}/2 + \frac{2}{5}H(\alpha, \zeta)\}, \quad \text{(P6)}$$

where $\widetilde{m}^i_{1950}$ and $\widetilde{m}^i_{2000}$ are i-th components of the vectors containing the data targets for the years 1950 and 2000. Observe that the value of the objective function from (P5) has been added to (P6).[19] Hence, the minimization procedure takes into account how changes in α and ζ influence the model's ability to match time allocations.[20] Due to the heavy time costs of simulating the full model, the parameter α was arbitrarily restricted to lie in a 21-point discrete set $\mathcal{A} = \{0.2, \cdots, 0.278, \cdots, 0.4\}$.[21]

[19] Note that there are five data targets in total, three involving vital statistics and two concerning time allocations. Also, observe that there are two observations for each of the vital statistics.

[20] Observe from (P5) that the parameters $\mathfrak{c}$, p_{1950}, and γ are functions of α and ζ.

[21] This set contains the value of α calibrated by Greenwood and Hercowitz (1991) for a business cycle model that includes household production and does not put leisure into the utility function. It also contains the value suggested by Cooley and Prescott (1995) for the standard real business cycle model. The results obtained are not that sensitive to the choice of α. Hence, this restriction does not seem that severe. Also, the spacing between the points in $\mathcal{A}$ is not linear. The set is refined over 3 successive iterations so that the points are clustered the closest around the optimal solution $\alpha = 0.278$.

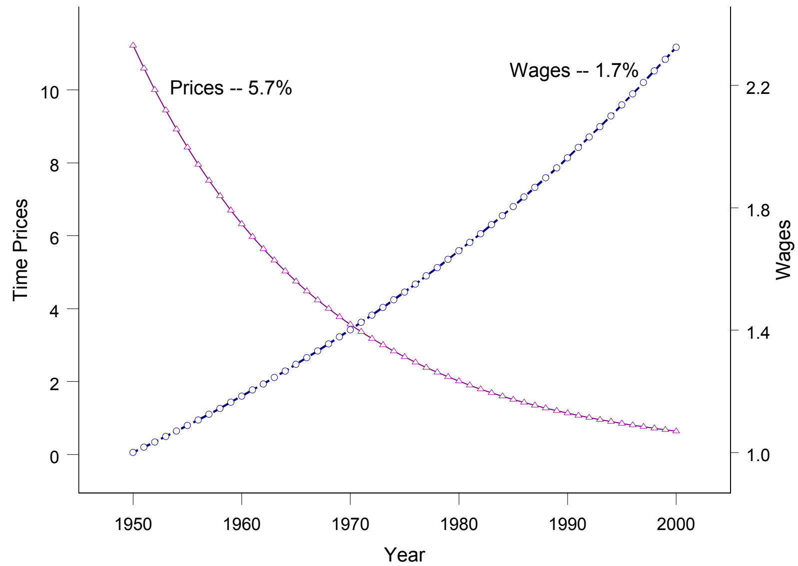

Figure 5: Wages and Prices, 1950-2000 – Model Inputs.

6.2 Results

Visualize the economy in 1950. Wages are low and the price for purchased household inputs is high, at least relative to 2000. Over time wages grow and the price for purchased household inputs falls. The time paths for wages and prices inputted into the analysis are shown in Figure 5. As can be seen, in the U.S. data wages increase 2.3 times over the time period in question. Prices are estimated to decline by a factor of 18. This seems large, but it is merely the result of compounding a 5.7 percent annual decline over a 50-year period. Can these two facts help to explain the decline in marriage and the rise in divorce over the last 50 years? This is the question asked here.

6.2.1 Household Hours

The time path for household hours that arises from the model is shown in Figure 6. It mimics the U.S. data reasonably well. In particular, the model matches very well the sharp increase in the fraction of time devoted to market work by married households. This is due to the

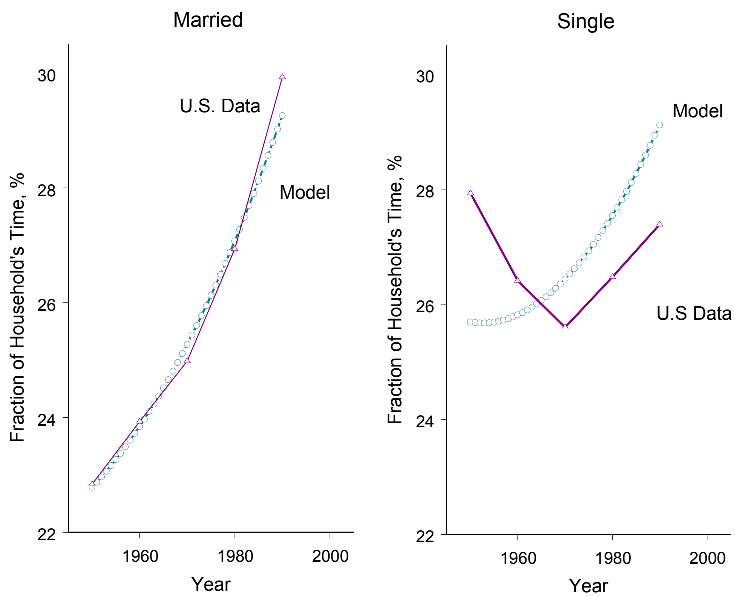

Figure 6: Household Hours, 1950-1990 – U.S. Data and Model.

declining price for purchased household inputs. Purchased household inputs and housework are substitutes in household production. As the price of purchased household inputs declines, households substitute away from using labor at home toward using goods. The model has trouble mimicing the enigmatic U-shaped pattern for single households. Still, it does a reasonable job at predicting the rise in participation from 1970 on. Observe that in 1950 married households devoted a smaller fraction of their time to market work than did single ones, both in the data and model. In the model this derives from the fixed cost of household maintenance. This forces low-income households to work more than high-income ones. In the model the low-income households are singles. As wages rise this effect disappears. By 1990 in the U.S. married households worked more than singles ones did. This is surprising since married households are much more likely to have children. In the model, they work about the same. Perhaps, in the real world, more productive individuals are also more desirable on the marriage market. Indeed, Cornwell and Rupert (1997) provide evidence that this is the case. Such a marriage-selection effect is missing in the model.

6.2.2 Vital Statistics

Now, the model starts off from an initial steady state that resembles the U.S. in 1950 and converges to a final one looking like the U.S. in 2000. In 1950 about 81.6 percent of the female population was married (out of non-widows who were between were the ages of 18 to 64). There were 10.6 divorces per 1000 married females, and 211 marriages. According to Schoen (1983) marriages lasted about 30 years in 1950. In 2000 the picture was quite different. Only 62.5 percent of females were married. The divorce rate had risen to 23 divorces by 1995, and the marriage rate had declined to about 80 marriages.[22] Finally, the average duration of marriages was about 20 to 24 years.[23] Table 3 shows the model's performance along these dimensions. Note that singles face a distribution with a low mean and a high variance, while married people face a distribution that has relatively a high mean, low variance, and high autocorrelation – see Table 2. This has two effects. First, it encourages singles to wait a while until a good match comes along. Second, it generates the long durations of marriages observed in the data.

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[22] Divorce and marriage statistics from the National Center for Health Statistics are not available after 1996.

[23] There are not any recent estimates for the duration of marriages. Schoen and Standish (2001) estimate that the duration of marriages to be about 24 years in 1995, while Espenshade (1985) estimates it to be 22.5 years for white females and 14.6 for black females over the period 1975-1980.

The steady-state duration of marriages in the model is given by

$$d_m = \frac{1}{1 - \pi_{mm}(1-\delta)},$$

where π_{mm} is the probability of a married agent remaining married next period.

TABLE 3: THE INITIAL AND FINAL STEADY STATES

	1950		*2000*	
	Model	Data	Model	Data
Fraction married	0.819	0.816	0.698	0.625
Probability of divorce	0.011	0.011	0.025	0.023
Probability of marriage	0.130	0.211	0.100	0.082
Duration of marriages	31.64	29.63	22.04	20-24

The fraction of the population that is married declines with the passage of time in the model. Figure 7 compares results obtained from the model with the U.S. data. The model can explain 12 percentage points of the observed 19 percentage point decline in the number of married females. This seems reasonable since other things went on in the world, such as a rise in the number of people going to college, a decline in fertility, etc. Observe that the utility differential between married and single life declines over time.[24] ,[25] This occurs for two reasons. First, recall that the utility function for nonmarket goods is more concave than the one for market goods. Thus, high-income households (married couples) spend less on household inputs relative to market consumption than do low-income household (singles). As a consequence, a fall in the price of purchased household inputs has a bigger impact on singles vis à vis married couples. Second, as wages rise the importance of the fixed cost for household maintenance disappears. This is more important for single households than married ones. Finally, many couples choose to live together but not marry. The framework can be thought of as modelling couples living together. The fraction of females living with a male fell by 16 percentage points between 1960 and 2000.[26] From this angle, the model

[24] In line with the discussion surrounding Lemma 5 the compensating differential needed to make a single as well off as a married person only falls from 20.4 percent to 18. This small decline is due to the fact that the fixed cost, $\mathfrak{c}$, is only a small fraction of the value of a single's time endowment, w.

[25] Cho and Siow (2003) estimate a non-transferable utility model of the U.S. marriage market. Their estimates show that the gains to marriage for young adults fell sharply between 1971 and 1981.

[26] The fraction of females living with a male is defined to be the fraction of females who are married plus the fraction of females who are unmarried living with a male. The size of this latter group is tabulated using

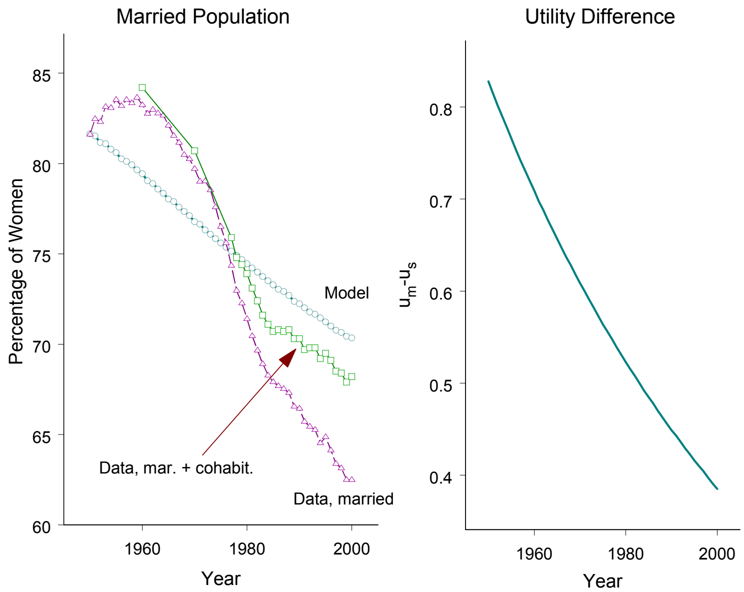

Figure 7: The Decline in Marriage, 1950-2000 – U.S. Data and Model.

captures about three quarters of the decline between 1950 and 2000.[27]

Underlying the decline in the fraction of the U.S. population that is married is a rise in the divorce rate and a decline in the rate of marriage. This is true for the model too, as can be seen in Figure 8. In the model divorces rise from 11 to 25, per 1,000 married women. This compares with 11 to 23 in the data. Marriages, in the model, fall from about 130 to 100 per 1,000 unmarried women. In the data they dropped from 141 to 69 or from 211 to 82, depending on the measure preferred. Thus, by either measure, the drop in marriages in the model is a little anemic. Again, it is not surprising that the model does not do well in this regard. Some important factors have been left out, such as the rise in education that surely must be associated with the delay in first marriages, or a narrowing in the gender gap that may have promoted female labor-force participation and made single life a more

the Census Bureau's 'posslq' household variable – persons of the opposite sex living together. This variable unfortunately also includes people who aren't partners. Still, it probably is a good proxy for the number of cohabitations.

[27] Note that the number of unmarried couples living together before 1960 would have been small and can be safely ignored.

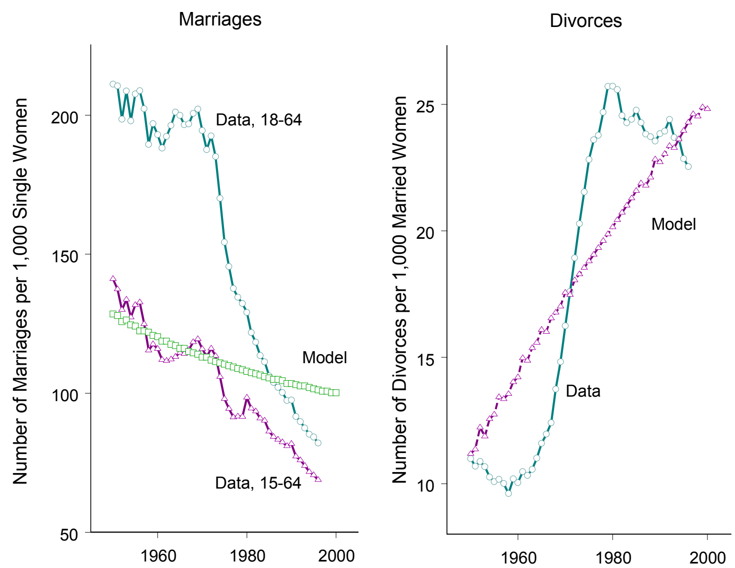

Figure 8: Rates of Marriage and Divorce, 1950-1996 – U.S. Data and Model

desirable option for females.[28] Last, in the data the duration of a marriage was 30 years in 1950. By 2000 this had declined to 20 years. The model does well in this regard. It predicts that duration of a marriage was 32 years in 1950 and 22 years in 2000.

7 Conclusions

The fraction of adult females who are married has dropped by roughly 20 percentage points since World War II. Females now spend a much smaller part of their adult life married than 50 years ago. Associated with this has been a rise in the divorce rate and a decline in the rate of marriage. At the same time, hours worked by married households rose considerably. This was driven by a large increase in labor-force participation by married females.

An explanation of these facts is offered here. The story told focuses on technological progress in both the household and market sectors. The idea is that investment-specific

[28] Regalia and Rios-Rull (2001) show that the decline in the gender gap played a significant role in the rise of single households during the 1970-1990 period.

technological progress in the household sector reduced the need to use labor at home. This simultaneously allowed women to enter the labor force and eroded the economic incentives for marriage. The analysis blends together a search model of marriage and divorce with a model of household production. The economic incentives for marriage derive from economies of scale in household production. These are whittled away overtime for two reasons. First, rising wages make it easier to meet or exceed the fixed cost for household maintenance. This reduces the need to marry to make ends meet. Second, a falling price for labor-saving household inputs has a bigger impact on single vis à vis married households, since the former devote a larger share of their spending to these products due to a high rate of diminishing marginal utility for nonmarket consumption. These two effects increase the (relative) value of single life.[29]

So, where can the analysis go from here? Technological progress in the home and market may affect the pattern of matching in society. There is some evidence that the degree of assortative mating in the U.S. has increased since 1940.[30] Extensions of the model may be able to capture this. Suppose that individuals differ in their labor market productivities. Assume that married males devote all of their time to market work while married females split their time between market work and household work. Now, when choosing a potential mate their earnings on the labor market will be a consideration. This will matter less at early stages of economic development, since married women will do little market work due to the large amount of time spent in household production. As women start to work more in the market, due to technological progress, it will begin to matter more. As an economy advances and the benefits from economies of scale in household consumption diminish, earnings potential along with marital bliss will become more important criteria when choosing a mate. The

[29] The economic forces that reduce the relative benefit of single versus married life may also have affected other living arrangements, such as the incentives of the elderly to live with their kids. Between 1970 and 1990 the fraction of widows living alone rose from 52.1 to 64.2 percent. Bethencourt and Rios-Rull (2004) argue that the rise in the relative income of elderly widows can account for a significant part of the rise in the number of elderly widows living alone between 1970 and 1990.

[30] See Lam (1997) for some facts on the correlation of income levels across partners and Mare (2000) for education.

degree of assortative mating will increase. Additionally, such an analysis would likely imply that the drop in the marriage rate should be biggest for those individuals in lower income groups, since the relative benefits from marriage will fall the most for them. Indeed, there is some evidence suggesting that this been has been the case.[31]

8 Appendix: The Impact of Technological Progress on Marriage and Divorce

The impact of technological progress on marriage and divorce will now be addressed. To this end, note that by standard arguments [Stokey and Lucas with Prescott (1989, chaps. 4 and 9)] it can be shown that the married value function V is strictly increasing in b. Thus, there is a unique value for b, or threshold t, that solves the equation $V(b) = W$. Furthermore, $V(b) \gtreqless W$ as $b \gtreqless t$. This allows the dynamic programming problems (P1) and (P2) to be rewritten more simply as

$$W = u^s + \beta \int^{t'} W' dS(b') + \beta \int_{t'} V'(b') dS(b'),$$

and

$$V(b) = u^m + b + \beta \int^{t'} W' dM(b'|b) + \beta \int_{t'} V'(b') dM(b'|b),$$

where again $u^s = I(1, p, w)$ and $u^m = I(2, p, w)$.

It is now easy to show that a purely temporary decrease in the price for purchased household inputs ($dp < 0$ with $dp' = dp'' = \cdots = 0$) will make individuals choosier about their mates.

Lemma 6 *A temporary one-shot decline in the price for purchased household inputs, p, will cause the threshold, t, to rise.*

[31] See Wallace (2000) who finds that the decline in the marriage rate is inversely related to the level of education.

Proof. The above equations imply

$$\begin{aligned} t &= u^s - u^m + \beta\int^{t'} W' dS(b') + \beta\int_{t'} V'(b')dS(b') \\ &\quad -\beta\int^{t'} W' dM(b'|b) - \beta\int_{t'} V'(b')dM(b'|b). \end{aligned}$$

Now note that W', V' and t' are unaffected by a purely temporary price decrease. Therefore,

$$\frac{dt}{dp} = \frac{d(u^s - u^m)}{dp} < 0,$$

by Proposition 4. Thus, a temporary decline in price will cause the threshold, t, to rise. ■
Since all individuals become more pickier about their mates, the rate of marriage will suffer a one period fall and the rate of divorce endure a one period increase.

To say much more, it looks like either some additional structure needs to be imposed on the framework, or that the model needs to be simulated numerically. The latter strategy is pursued in Section 6. Following the first strategy, the stochastic structure of the model will be simplified in way that is commonly done with search models – for example, see Wright and Loberg (1987).

Assumption *Let the stochastic structure governing match quality be specified as follows:*
(i) For single agents the cumulative distribution function S has bounded support with upper bound $\overline{b}$;
(ii) For married agents let b evolve in line with

$$b' = \begin{cases} = b, & \text{with } \Pr(\pi), \\ = \underline{b}, & \text{with } \Pr(1-\pi), \end{cases}$$

where $\underline{b} < [U^s(w, \overline{p}) - U^m(w, \overline{p})\ ]$ with $\overline{p}$ representing the upper bound on the price for household products. The lower bound $\underline{b}$ is constructed so that it is never optimal to marry given this match quality.

Given this, the dynamic programming problems for single and married agents, in a stationary equilibrium, will appear as

$$W = u^s + \beta\int_t V(b)dS(b) + \beta W S(t), \tag{14}$$

and

$$V(b) = u^m + b + \pi\beta V(b) + (1-\pi)\beta W. \tag{15}$$

Lemma 7 *A decrease in the price for purchased household inputs, p, causes the steady-state threshold level of match quality, t, to rise.*

Proof. An equation characterizing the threshold, t, will be derived. To this end, first rewrite (15) as

$$V(b) = \frac{u^m + b + (1-\pi)\beta W}{1-\beta\pi}.$$

Now, use this in (14) to obtain an equation defining W:

$$[1 - \beta\pi - \beta^2(1-\pi) - \beta S(t)(1-\beta)]W = (1-\beta\pi)u^s + \beta \int_t (u^m + b)dS(b).$$

Note that (15) can also be evaluated at $b = t$ to get $W = [u^m + t]/(1-\beta)$. This allows the above equation to be converted into a condition specifying t. Specifically, solving out for W gives

$$[\frac{1 - \beta\pi - \beta^2(1-\pi)}{1-\beta} - \beta S(t)](u^m + t) = (1-\beta\pi)u^s + \beta \int_t (u^m + b)dS(b),$$

which can be rearranged to read[32]

$$[1 + \beta(1-\pi) - \beta S(t)](u^m + t) = (1-\beta\pi)u^s + \beta \int_t (u^m + b)dS(b).$$

This equation determines t. Integrating by parts the right-hand side of the above condition yields

$$[1 + \beta(1-\pi) - \beta S(t)](u^m + t) = (1-\beta\pi)u^s + \beta u^m[1 - S(t)] + \beta\overline{b} - \beta t S(t) - \beta \int_t^{\overline{b}} S(b)db,$$

which can be rewritten as

$$[1 + \beta(1-\pi)]t = (1-\beta\pi)(u^s - u^m) + \beta\overline{b} - \beta \int_t^{\overline{b}} S(b)db.$$

[32] Note that $1 - \beta\pi - \beta^2(1-\pi) = (1-\beta)[1 + \beta(1-\pi)]$.

Finally, differentiate the above equation to get

$$\frac{dt}{dp} = \frac{(1-\beta\pi)}{1+\beta(1-\pi)-\beta S(t)} \frac{d(u^s - u^m)}{dp} < 0,$$

where the sign follows from Proposition 4. Hence, a decrease in price, p, will increase t. ■

So, how will a decrease in the steady-state price for purchased household inputs, p, affect the number of people who are married in a steady state? To answer this, let χ represent the steady-state fraction of people who are married and s denote the density function that is associated with S. The fraction of people who are married in a steady state is determined by the equation

$$\chi = (1-\chi)[1-S(t)] + \chi(1-\pi),$$

so that

$$\chi = \frac{1-S(t)}{1+\pi-S(t)}.$$

From this it is immediate that

$$\frac{d\chi}{dp} = \frac{-\pi}{[1+\pi-S(t)]^2} s(t) \frac{dt}{dp} > 0,$$

by Lemma 7. Thus, a fall in price will lead to a decrease in the fraction of the population that is married.

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