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On the rising tendency of disability risk in the process of mortality decline

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Reducing mortality is widely taken as a social development target; its side effects have rarely been mentioned. On the basis of multistate model, this paper introduces necessary measures, and illustrates that mortality decline raises disability risk. This effect implies living longer does not lead to living healthier, and requires thinking about how to reduce disability risk. To show the effect of mortality decline raising disability risk in a controllable way, this paper simulates consequences of hypothetical mortality declines, using the US data.

Living healthier is often accompanied by living longer, but living longer may neither come from nor lead to living healthier. Common people could tell the difference between living longer and healthier from observing someone was sick in bed long before death while others did not, and demographers employ life expectancy (LE) and active life expectancy (ALE) measures to gauge the difference between simply living longer and living longer in robust health.

Several decades ago, Sullivan (1971) introduced a calculation for ALE, which is now termed the *prevalence method*. In this method, age-specific death rates (denoted as m(x) at age x) are used to produce an ordinary life table. The person-years lived in each age group is then divided into active and disabled statuses according to the disabled proportion (d(x) at age x) which can be found in census data from many countries (see Weeks, 1999). Subsequently, the total-person-years over each age are obtained for both active disabled statuses. Dividing the active total-person-years over a certain age by the number of survivors at this age, the result is the prevalence ALE, and the difference between LE and ALE is therefore the disabled life expectancy (DLE).

Distinguishing ALE from LE is a significant progress, especially since rising LE is widely interpreted as socioeconomic success (e.g., the human development index, www.undp.org/report). Examining the US trends in the 1970s, Crimmins, Saito and Ingegneri (1989) showed that the most of increase in LE was DLE, and thus provided perhaps the first national-level evidence that living longer does not always mean living healthier. Consequently, monitoring the change of ALE is becoming a demographic routine (e.g., Crimmins, Saito and Ingegneri, 1997; Cambois, Robine and Hayward, 2001), and ALE is included among national health goals of the US (Office of Disease Prevention and Health Promotion. 1991).

Problems of the prevalence ALE and DLE, however, should not be ignored. In specific, the disabled proportion (d(x)) does not measure the disability risk at the time of corresponding census, because disabled individuals aged x were disabled not only at census but also earlier times. Thus, comparing prevalence ALE between times or regions is problematical. Furthermore, the relationship between mortality and disability cannot be discussed since the fundamental measure of disability risk, which should be independent of mortality, is not yet defined. Therefore, although the effect of mortality decline raising disability risk could be vaguely sensed in occasions where DLE increased faster than did LE, this effect cannot be investigated in a controllable way.

The fundamental measure of disability risk and the multistate stationary population

Before defining the fundamental measure of disability risk, the purpose and data availability should be discussed. In order to compare disability risks between times, cross-section data are relevant. Besides, cross-section data are likely to be available at national level. Limited to cross-section data, an individual's information, such as gender and age as well as whether or not disabled, could refer only to a 'current' state. In this situation, a disabled individual's return to active status is canceled by an active person becoming disabled at the same age, because whether an active person was in disabled status prior to the 'current' state cannot be identified. Thus, it is the number of net transitions from active to disabled status, defined by the number of active-to-disabled transitions minus the number of disabled-to-active transitions, that should be modeled.

The fundamental measure of disability risk can be defined in the way similar to age-specific death rate if the onset of disability is viewed as analogical to death. Dividing the number of active-to-disabled net transitions in age group (x,x+1] in a calendar year by the average number of active population in this age group and year, the result is the *active-to-disabled net transition rate*, which is denoted as t(x). The t(x) could be obtained from census, if since when a disabled person was disabled were asked. Before such ideal information becomes available, t(x) can also be estimated through well examined assumptions (e.g., Li, 2004). The t(x) measures the risk of becoming disabled at age x under the condition of surviving to age x in active status, and is independent of mortality. As a vector, however, t(x) is incomparable between different times and regions. For example, comparing to an earlier period, the value of t(x) in a later period may be larger at a younger age but smaller at an older age, and thus we cannot evaluate which period has the higher disability risk. This problem is similar to evaluating mortality when age-specific death rates compose an incomparable vector, and the stationary population is used as a solution.

To model a population that includes two status, death rates of each status, namely $m_a(x)$ for active and $m_d(x)$ for disabled, need to be introduced. Similar to t(x), $m_a(x)$ and $m_d(x)$ could be obtained if the active or disabled status of each death in vital statistics were identified by matching with what is record in earlier census. Before such matching is made, $m_a(x)$ and $m_d(x)$ can also be estimated through well examined assumptions (e.g., Li, 2004).

For convenience, at any time and at the starting age s, the numbers of active and disabled population can be set as [1-d(s)] and d(s), respectively. Assuming that t(x) and $m_a(x)$ are constant over time and across age group (x, x+1], and viewing

$[t(x)+m_a(x)]$ as the 'death rate', the stationary state of active population, with $l_a(x)$ survivors at age x, will be reached from any initial state in the way similar to reaching an ordinary stationary population:

$$l_a(x+1) = l_a(x)\exp\{-[m_a(x)+t(x)]\}. \quad (1)$$

After the active population becomes stationary, the number of becoming disabled in a subinterval of (x, x+1], namely (z, z+dz], is constant over time and can be written as $t(x)l_a(z)dz$. The number of their surviving to age (x+1) is therefore $t(x)l_a(z)\exp[-m_d(x)(1-z)]dz$. Thus, if the disabled population at ages younger than x became stationary at a certain time, then at the subsequent time the number of survivors aged (x+1) will become stationary and can be written as

$$l_d(x+1) = l_d(x)\exp[-m_d(x)] + \int_0^1 t(x)l_a(z)\exp[-m_d(x)(1-z)]dz$$

$$= l_d(x)\exp[-m_d(x)] + t(x)l_a(x)\frac{\exp\{-[m_a(x)+t(x)]\} - \exp[-m_d(x)]}{m_d(x)-m_a(x)-t(x)}. \quad (2)$$

Because at the starting age the disabled population is stationary as d(s) since the initial time, disabled population will eventually reach stationary at any age. Subsequently, the person-years lived in age group (x, x+1], namely $L_a(x)$ for active and $L_d(x)$ for disabled status, the stationary disabled proportion $d_o(x)=L_d(x)/[L_a(x)+L_d(x)]$, and the stationary death rate of total population $m_o(x)=d_o(x)m_d(x)+[1-d_o(x)]m_d(x)$, are obtained.

For the oldest open age group including age w and older, however, (2) does not apply, and $L_d(w+)$ cannot be derived directly. Nonetheless, for active status, $L_a(w+)$ can still be expressed as $1/[t(w+)+m_a(w+)]$, assuming that $t(w+)$ and $m_a(w+)$ are constant across all ages over w. Because $|m(x)-m_o(x)|=|[d_o(x)-d(x)][m_a(x)-m_d(x)]|$ are much smaller than $|m_a(x)-m_d(x)|$, $|m(x)-m_o(x)|$ should be small for all finite age groups. The difference between $m(w+)$ and $m_o(w+)$ should also be small, if $m_a(w+)$ and $m_d(w+)$ as well as $t(w+)$ could be assumed constant across age so the open age group can be treated as other age groups. Thus, $m_o(w+)$ can be replaced by the observed $m(w+)$, the person-years of the oldest open age group in the stationary total population can be written as,

$$L_o(w+) \approx [l_a(w)+l_d(w)]/m(x), \quad (3)$$

and the person-years of oldest open age group in the stationary disabled population is obtained as

$$L_d(w+) \approx L_o(w+) - L_a(w+). \quad (4)$$

The multistate stationary population is therefore established. The total-person-years over age s, namely $T_a(s)$ for active and $T_d(s)$ for disabled status, are sums of $L_a(x)$ and $L_a(x)$ from age s, respectively. The prevalence ALE and DLE can then be rebuilt on the basis of 'current' disability and mortality risks, called the stationary ALE and DLE, and denoted as $e_a(s)=T_a(s)/[l_a(s)+l_d(s)]=T_a(s)$ and $e_d(s)=T_d(s)$, respectively.

The stationary disabled proportion, $d_o(x)$, reached by fixing the 'current' t(x) and $m_a(x)$ as well as $m_d(x)$, may remarkably differ from the 'current' disabled proportion d(x) that is resulted from historical changes of mortality and disability. Nonetheless, the stationary death rate for the total population, $m_o(x)$, would differ only slightly from m(x) for the reason mentioned above. Consequently, the two stationary total populations, of which one is calculated from m(x) on the homogenous assumption that individuals aged x are identical and another is computed from $m_o(x)$ using the multistate model that divides individuals into two status, should differ only slightly.

Measures of accumulative disability risk

The fraction of disabled life expectancy, FDL(s), is often used to describe disability risk accumulated over age s. Now the FDL(s) stands on 'current' risks of disability and mortality, and can be compared over times and between regions. Because $FDL(s)=e_d(s)/[e_a(s)+e_d(s)]=e_d(s)/e(s)$, it indicates the fraction of disabled life expectancy for both active and disabled individuals aged s. Mixing up active and disabled individuals at age s, who face different risks at older ages, the FDL(s) cannot properly measure accumulative disability risk. For example, a higher FDL(s) may result from a larger $l_d(s)$ and has nothing to do with higher disability risk over age s.

Because the multistate model is established, accumulative disability risk can be measured one way or another. In this model the equilibrium of each status is described, the question is how to compose aggregate indexes to describe the risk of entering and staying in a specific status: disabled. Since it is the active-to-disabled net transition that should be modeled, the risk of becoming disabled at ages older than s is relevant only for active people aged s, and this risk can be measured as following.

Active individuals aged s may either die or become disabled at older ages. In the active stationary population and the age group (x, x+1], the number of becoming disabled is $t(x)L_a(x)$. The lifetime disable probability for active survivors aged s, namely LDP(s), is therefore obtained as

$$LDP(s) = \frac{1}{l_a(s)} \sum_{x=s}^{w+} t(x) L_a(x). \quad (5)$$

The LDP(s) measures the risk of becoming disabled at ages older than s for active people aged s. According to (1), reducing active mortality at age x causes $L_a(y)$ to increase at y>x, which in turn raises LDP(s) through (5), if t(x) were unchanged. Thus, decline of active mortality at ages older than s raises the risk of becoming disabled over age s. Equation (5) also illustrates that changing mortality of disabled status does not affect the risk of becoming disabled. Therefore, any mortality decline that includes active mortality raises the risk of becoming disabled, if t(x) remain unchanged.

Without utilizing mortality of disabled status, LDP(s) excludes the risk of staying in disabled status. For example, LDP(40)=0.5 indicates that half of active individuals aged 40 will eventually become disabled. But this does not describe at what ages these active people become disabled, which may determine how long to live in disabled status and should also be discussed.

Let $L^*_d(x)$ be the disabled-person-years in age group (x, x+1] that is composed only by disabled people aged s<x, and $T^*_d(s)$ and $e^*_d(s)=T^*_d(s)/l_d(s)$ be the corresponding total-person-years and life expectancy. In other words, $L^*_d(x)$ and $T^*_d(s)$ as well as $e^*_d(s)$ are quantities in the absence of active-to-disabled net transition at ages over s. Then $[T_d(s)-T^*_d(s)]$ is the total-person-years due to active-to-disabled net transitions over age s, and $[T_d(s)-T^*_d(s)]/l_a(s)$ is the disabled life expectancy of active individuals aged s. As disabled life expectancy of active people, $[T_d(s)-T^*_d(s)]/l_a(s)$ reflects both the risk of entering and staying in disabled status and the level of disabled mortality, and should not be used as a measure of only the former. This is because, if a population's $e^*_d(s)$ is higher than another, its $[T_d(s)-T^*_d(s)]/l_a(s)$ would also be higher, even if its age pattern of becoming disabled is the same as another population. This is similar to the situation of FDL(s), in which $e_d(s)$ reflects both disability risk and overall mortality, and therefore the $FDL(s)=e_d(s)/e(s)$, instead of $e_d(s)$ itself, is often used to measure disability risk over age s. Unlike that $e_d(s)$ is a fraction of e(s), however, the relationship between $[T_d(s)-T^*_d(s)]/l_a(s)$ and $e^*_d(s)$ needs further analysis.

The $[T_d(s)-T^*_d(s)]$ is composed by individuals who are active at age s, become disabled later at different ages and stay in disabled status for different years. How to simplify this complex process? I proposed a procedure (Li, 2004) that begins from recalling the strategy of life expectancy at age s, e(s). For individuals aged s, they will die at older but different ages. The basic idea of life expectancy is to ask if the total-person-years over age s maintained invariant and all individuals died at the same age, what would this age be? The answer is [s+e(s)] years.

Now there are $l_a(s)$ active individuals aged s, some of them will become disabled before death, and the total-person-years of their living in disabled status is $[T_d(s)-T^*_d(s)]$. Similar to the situation of life expectancy, I ask if $[T_d(s)-T^*_d(s)]$ remained invariant and all active individuals who become disabled did so at the same age s, what would the number of these individuals be? This number must be a fraction of $l_a(s)$. Defining this fraction as the *equivalent disability fraction* at age s and denoting it as EDF(s), the number of those who become disabled hypothetically at age s is $EDF(s)l_a(s)$. Since these active individuals become disabled at age s, they will follow disabled mortality then and live $e^*_d(s)$ years in disabled status, and the total-person-years of living in disabled status due to this hypothetical transition is

$EDF(s)l_a(s)e^*_d(s)$. Because the total-person-years of living in disabled should hold constant, there is

$$EDF(s) = \frac{T_d(s) - T_d^*(s)}{l_a(s)e_d^*(s)}. \quad (6)$$

The basis of EDF(s) is the equivalence of disabled total-person-years. On one hand, the number of active persons who become disabled at different ages older than s is not $EDF(s)l_a(s)$ but larger. On the other hand, the years of living in disabled status for each of such person is not $e^*_d(s)$, but fewer. If two persons living in disabled status for one year were equivalent to one person living in disabled status for two years, the different ages at which active individuals become disabled can be simplified as one age that is s, and the different years in which they live in disabled status can be summarized as a unique number which is $e^*_d(s)$. This simplification requires a certain number of active individuals, $EDF(s)l_a(s)$, to become disabled, and provides a measure of risk of becoming and staying in disabled statues over age s, EDF(s).

The EDF(s) can be explained in the way similar to that of FDL(s). The $[T_d(s)-T^*_d(s)]/l_a(s)$ would be higher when the disabled mortality is lower and vice versa. Dividing $[T_d(s)-T^*_d(s)]/l_a(s)$ by $e^*_d(s)$ reduces the effect of disabled mortality level, and leads to EDF(s). The EDF(s) has also the following geometric interpretation. The $[T_d(s)-T^*_d(s)]$ measures the area under the curve $[L_d(x)-L^*_d(x)]$ for x>s, which equals that of the square with width $l_a(s)e^*_d(s)$ and height EDF(s). Subsequently, $[T_d(s)-T^*_d(s)]$ is divided into two orthogonal components: one is $l_a(s)e^*_d(s)$ which describes the active population size and disabled mortality level, and another is EDF(s) that measures the risk of entering and staying in disabled status over age s, controlling for $e^*_d(s)$.

For given t(x), reducing active mortality raises $T_d(s)$ but does not affect $T^*_d(s)$. Accordingly, (6) indicates that reducing active mortality raises EDF(s). Lowering disabled mortality raises $T_d(s)$ and $T^*_d(s)$ as well as $e^*_d(s)$. Comparing to reducing active mortality, the effect of lowering disabled mortality would be smaller, because the increases of both numerator and denominator of EDF(s) will cancel each other. The up or down direction in the change of EDF(s), however, depends on the age patterns of mortality change and becoming disabled. Consequently, when active and disabled mortality decline by similar rates, EDF(s) will increase.

Examples and discussion

I use the US data in 1990 for examples. The age-specific death rates are adopted from Tuljapurkar, Li and Boe (2000). The disabled proportions, d(x), are cited from Crimmins, Saito and Ingegneri (1997), in which a disabled person is defined as not being able to perform the normal activities of life including going to school for children, working, keeping house or other things that people do. These data are listed in the second and seventh columns in Tables 1 and 2, respectively. The $m_a(x)$, $m_d(x)$ and t(x) are not yet available at national level. Based on the well-known Gompertz exponential law (Gompertz, 1825) and the Cox proportional hazard model (Cox, 1972), I proposed a procedure (Li, 2004) that estimates the values of $m_a(x)$, $m_d(x)$ and t(x) as in shown in Tables 1 and 2. The $m(x)=[1-d(x)]m_a(x)+d(x)m_d(x)$ are close to the observed values of age-specific death rates, implying that the estimated $m_a(x)$, $m_d(x)$ and t(x) should be close to their true but unknown values. Since the purpose of this paper is not to discuss the accuracy of these estimates, I assume that the values of m(x), $m_a(x)$, $m_d(x)$ and t(x) are calculated directly from census and vital statistics as they could be, and call them observed values.

Table 1. Mortality and disability data of the U.S. males, 1990

Age x	Observed death rates	$m_a(x)$	$m_d(x)$	t(x)	$d_o(x)$	d(x)	$m_o(x)$	m(x)
40-44	0.0034	0.0033	0.0036	0.0091	0.1539	0.1345	0.0033	0.0033
45-49	0.0049	0.0050	0.0055	0.0110	0.1960	0.1707	0.0051	0.0051
50-54	0.0076	0.0076	0.0084	0.0142	0.2460	0.2128	0.0078	0.0078
55-59	0.0121	0.0116	0.0129	0.0217	0.3121	0.2635	0.0120	0.0120
60-64	0.0190	0.0178	0.0197	0.0229	0.3872	0.3370	0.0185	0.0184
65-69	0.0285	0.0272	0.0301	0.0292	0.4639	0.4060	0.0285	0.0284
70-74	0.0435	0.0416	0.0460	0.0287	0.5387	0.4909	0.0440	0.0438
75-79	0.0661	0.0636	0.0703	0.0307	0.6035	0.5686	0.0676	0.0674
80-84	0.1014	0.0972	0.1074	0.0413	0.6690	0.6577	0.1041	0.1039
85+	0.1655	0.1486	0.1642	0.0762	0.7963	0.8006	0.1611	0.1611

Table 2. Mortality and disability data of the U.S. females, 1990

Age x	Observed death rates	$m_a(x)$	$m_d(x)$	t(x)	$d_o(x)$	d(x)	$m_o(x)$	m(x)
40-44	0.0016	0.0016	0.0017	0.0076	0.1563	0.1401	0.0016	0.0016
45-49	0.0027	0.0026	0.0027	0.0105	0.1942	0.1708	0.0026	0.0026
50-54	0.0043	0.0041	0.0042	0.0089	0.2333	0.2115	0.0041	0.0041
55-59	0.0068	0.0065	0.0067	0.0087	0.2670	0.2420	0.0066	0.0066
60-64	0.0107	0.0104	0.0106	0.0149	0.3097	0.2684	0.0104	0.0104
65-69	0.0160	0.0164	0.0168	0.0262	0.3791	0.3168	0.0166	0.0165
70-74	0.0246	0.026	0.0267	0.0311	0.4666	0.4055	0.0263	0.0263
75-79	0.0386	0.0413	0.0423	0.0499	0.5690	0.5040	0.0419	0.0418
80-84	0.0638	0.0655	0.0671	0.1066	0.7115	0.6539	0.0667	0.0666
85+	0.1145	0.1039	0.1065	0.2221	0.9385	0.9263	0.1063	0.1063

Assuming $m_a(x)$, $m_d(x)$ and t(x) are constant over time, the population will be stationary from any initial state including that associated with d(x) and m(x). As can be seen in Tables 1 and 2, the stationary $d_o(x)$ differ remarkably from d(x), which were observed in 1990. Nevertheless, as expected, the stationary $m_o(x)$ are almost identical to the m(x) observed in 1990, implying slight difference between the multistate-heterogeneous and ordinary-homogenous stationary population.

For examples in this paper, I take s=40, because the disability risk is low at ages younger than 40 years, and older than age 40 the Gompertz law works well. Based on data in Tables 1 and 2, FDL(40) and LDP(40) as well as EDF(40) are calculated as baseline values and shown in Table 3.

Table 3. The baseline measures of accumulative disability risk over age 40

	FDL(40)	LDP(40)	EDF(40)
Male	0.3788	0.5072	0.2897
Female	0.4091	0.6219	0.3144

For population aged 40, values of FDL(40) are 0.38 for males and 0.41 for females. Since the value of d(40) for males is also smaller than for females as can be seen in Tables 1 and 2, it is hard to tell which gender's disability risk is higher at ages over 40. This is because the smaller value of FDL(40) for males may be a result of fewer disabled individuals aged 40, and have nothing to do with disability risk at ages over 40.

In the stationary state of active population, the age-specific numbers of becoming disabled are shown in Figure 1. Summing these numbers over age and dividing by the number of active people aged 40, the values of LDP(40) are obtained as 0.51 for males and 0.62 for females. Apparently, the risk of becoming disabled is high, and for females it is 22% higher than for males. But, Figure 1 also shows that females become disabled at older ages than do males. Consequently, the risk of entering and staying in disabled status for females should not be so much higher, or could be even lower, than for males. But how much higher or lower?

Those who are active at age 40 would become disabled following the age patterns shown in Figure 1, survivor to different ages subjecting to disabled mortality, and eventually form the stationary age structure according to (2). These stationary age structures are shown by the solid curves in Figure 2 for males and Figure 3 for females. The disabled total-person-years for active people aged 40 is the area below the solid curve in Figure 2 or 3, whose value depends also on the disabled mortality that can be measured by $e^*_d(40)$. Among those who are active at age 40, the fraction of EDF(40) would have to become disabled at this age in order to yield a disabled total-person-years that equals the area under the solid curve. This hypothetical disabled total-person-years is measured by the area in the dashed square in Figure 2 or 3. And this equivalent square separates the disabled-total-person-years into two components: the width $l_a(40)e^*_d(40)$ that reflects the effect of active population size as well as disabled mortality level, and the height EDF(40) that measures the risk of entering and staying in disabled status.

The values of EDF(40) are 0.29 for males and 0.31 for females. Thus, the risk of entering and staying in disabled status for females is only 7% higher than for males, which is much lower than the 22% from comparing the values of LDP(40) and is expected from analyzing Figure 1.

In the model of EDF(40), individuals become disabled at the same age 40 that is younger than in that of LDP(40). Meanwhile, the disabled total-person-years of EDF(40) equals that would be derived by LDP(40). Therefore, the value of EDF(40) should be smaller than that of LDP(40), as shown in Table 1.

The measure of the risk of becoming disabled at age x under the condition of surviving to this age in active status, t(x), is independent with mortality change. Fixing t(x), effect of reducing mortality on accumulative disability risk can be examined. This effect would reflect the consequence of mortality decline, if reducing fundamental disability risk were ignored. In order to illustrate this effect in a controllable way, I use three scenarios of mortality decline: (1) $m_a(x)$ drops 10%, (2) $m_d(x)$ drops 10%, and (3) that both $m_a(x)$ and $m_d(x)$ drop 10%, at all x>s. Mortality has been observed dropping by similar rate at ages over 40 (Lee, Tuljapurkar and Li, 2004), and could be approximated by scenario (3) or other combinations of scenarios (1) and (2). To show the consequences on FDL(40) and LDP(40) as well as EDF(40), the ratio of their values in each scenario to that in baseline is displayed in Figure 4 for males and Figure 5 for females.

Reducing active mortality at ages older than 40 would lift $T_a(40)$ directly through (1) and also raise $T_d(40)$ because, as can be seen in (2), some of the increased active survivors would become disabled. On one hand, the increase of $T_d(40)$, due to some of the increased active survivors becoming disabled, should be smaller than that of $T_a(40)$ that is composed by the increased active survivors themselves. On the other hand, given the same increase of $T_a(40)$ and $T_d(40)$, the reduction of $FDL(40)=T_d(40)/[T_a(40)+T_d(40)]$ caused by the rising of $T_a(40)$ would be smaller than the increase of FDL(40) resulting from the rising of $T_d(40)$. As result, the effect of reducing active mortality on FDL(40) would be slight with uncertain up or down direction, as presented by consequences of scenario (1) in Figures 4 and 5.

Reducing disabled mortality at ages older than 40 lifts $T_d(40)$ but does not affect $T_a(40)$, and therefore raises FDL(40), as in scenario (2) in Figures 4 and 5. This indicates that reducing disabled mortality raises the risk of staying in disabled status, without controlling for $e^*_d(40)$. Since reducing active mortality would change FDL(40) only slightly, lowering both active and disabled mortality by the same rate, as in scenario (3), would raise FDL(40), as shown in Figures 4 and 5.

Because of mixing up active and disabled people, FDL(40) may mislead policy studies. For example, reducing active mortality and ignoring disabled people would not systematically raise FDL(40), as demonstrated by the consequences of scenario (1) in Figures 4 and 5. Furthermore, raising disabled mortality, though too inhuman to be even thought of, reduces FDL(40).

The reactions of LDP(40) are easy to understand. Reducing active mortality increases active survivors. Since some of the increased active survivors will become disabled, the number of becoming disabled, and hence LDP(40), increase, as shown by scenario (1) in Figures 4 and 5. Changing disabled mortality has nothing to do with how active people die and become disabled, and thus does not affect LDP(40). Accordingly, the consequences of scenarios (1) and (3) are identical, and any mortality decline that includes active mortality raises LDP(40).

As also expected from above analysis, scenario (1) shows that reducing active mortality raises EDF(40), scenario (2) indicates that lowering disabled mortality changes EDF(40) slightly with uncertain trend, and scenario (3) illustrates that reducing both active and disabled mortality by the same rate raises EDF(40), in Figures 4 and 5. Lowering disabled mortality, which is shown by scenario (2), would apparently raise the risk of staying in disabled status. Then why EDF(40) declines in Figures 4 and 5? This is because EDF(40) measures the risk of entering and staying in disabled status by controlling for the $e*_d(40)$. Although the years that active people aged 40 lived in disabled status increased, its increase is smaller than that of $e*_d(40)$. In other words, the years that active people aged 40 lived in disabled status, relative to that of disabled people aged 40, reduced in scenario (2).

If the task of reducing the fundamental disability risk t(x) were ignored, mortality decline would raise accumulative disability risk according to all the three measures, which were already high according to the indirect data from 1990. The values of LDP(40) in Table 3 imply that more than half of active people aged 40 would die in disabled status.

But is it true that most people are disabled shortly before death, so that the LDP should be close to 1 and does not make much sense? Yes, if the death and transition rates were associated to infinitively short period rather than one year. In this paper, active people can either die or become disabled, but cannot do both, in one year. Thus, those who become disabled and then die in one year are counted as died in active status. Since these people would live in disabled statues for half year on average, measuring death and transition rates in one year ignores short-than-half-year disabled statues, and the LPD in this paper includes longer-than-half-year disabled statues. Nonetheless, this is not a practical problem, because short-term disabled does not impose serious burden, emotionally or economically. For example, by definition many people are driven into disabled statues shortly by diseases and events such as flu or even hangover, but they are not regarded as disabled.

The LDP, however, does raise a practical question. Among the active people aged 40, more than half will become disabled at least half year before death. But being disabled for half or ten years before death is a big difference, how to measure the disabled duration? The EDF(40) answers. According to the definition of disability mentioned above, about 30% active people would have become disabled at age 40 in order for the disabled total-person-years to equal that generated by the accumulative disability risk. Imaging how high the disability risk would be if 30% active people became disabled at age 40 besides those who were already so at this age. Moreover, given that the declines of mortality at ages over 40 had been significant in the second half of the last century (Lee and Miller, 2001) and are likely to be so in the future, accumulative disability risk would be even higher in the future if sufficient effort of reducing t(x) were not made.

The consequences of reducing t(x) are simple. Through raising active proportion in the stationary population, reducing transition rates lowers m(x), if $m_a(x)<m_d(x)$. But, this process cannot make m(x) lower than $m_a(x)$. Therefore, reducing transition rates may lower m(x) only slightly, as can be seen in Tables 1 and 2. As to accumulative disability risk, $T_a(40)$ will increase and $T_d(40)$ will decline, but $T^*_d(40)$ will maintain unchanged. Thus, FDL(40) and EDF(40) will decline. Since lowering t(x) reduces the number of becoming disabled at age x, and since only part of this reduction will become disabled at older ages, so the total number of becoming disabled, and hence LDP(40), will decline. Therefore, according to all the three measures, accumulative disability risk can be reduced by lowering t(x). What factors affect and how to lower t(x), however, are issues to be further explored.

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Figure 1. Age patterns of becoming disabled

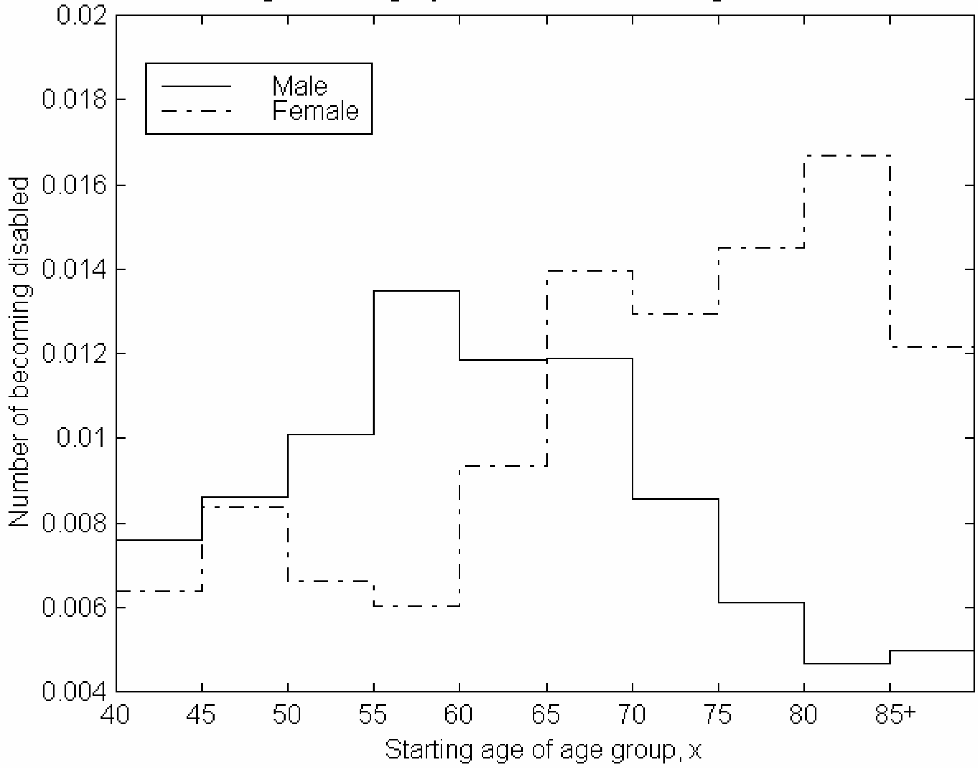


Figure 2. Equivalent disability fraction at age 40 for males

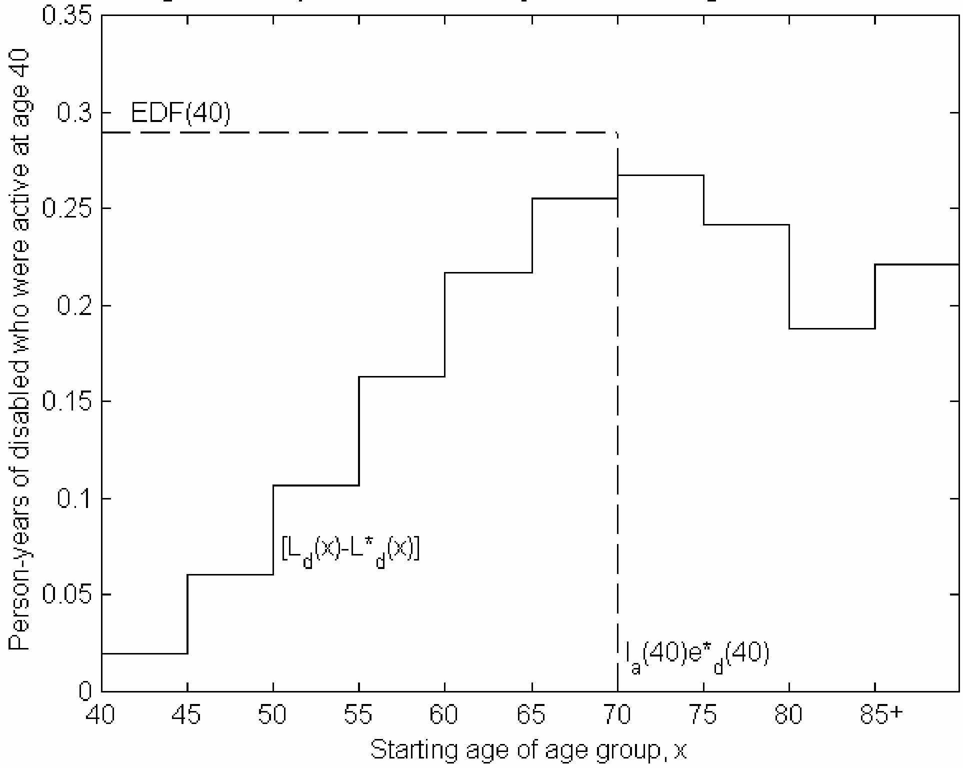


Figure 3. Equivalent disability fraction at age 40 for females

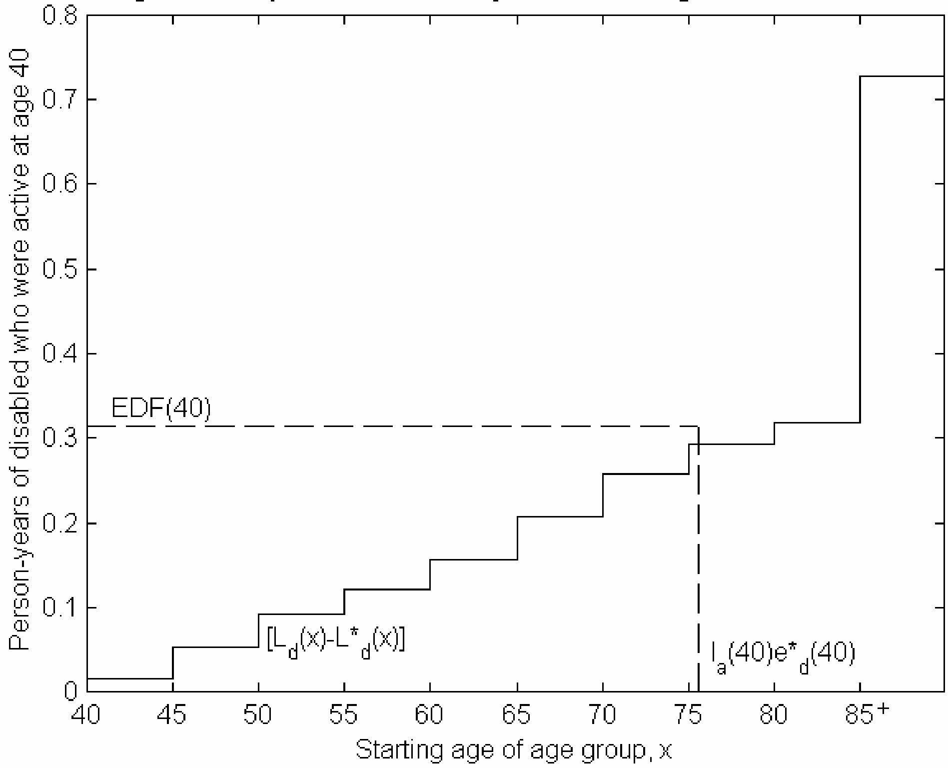
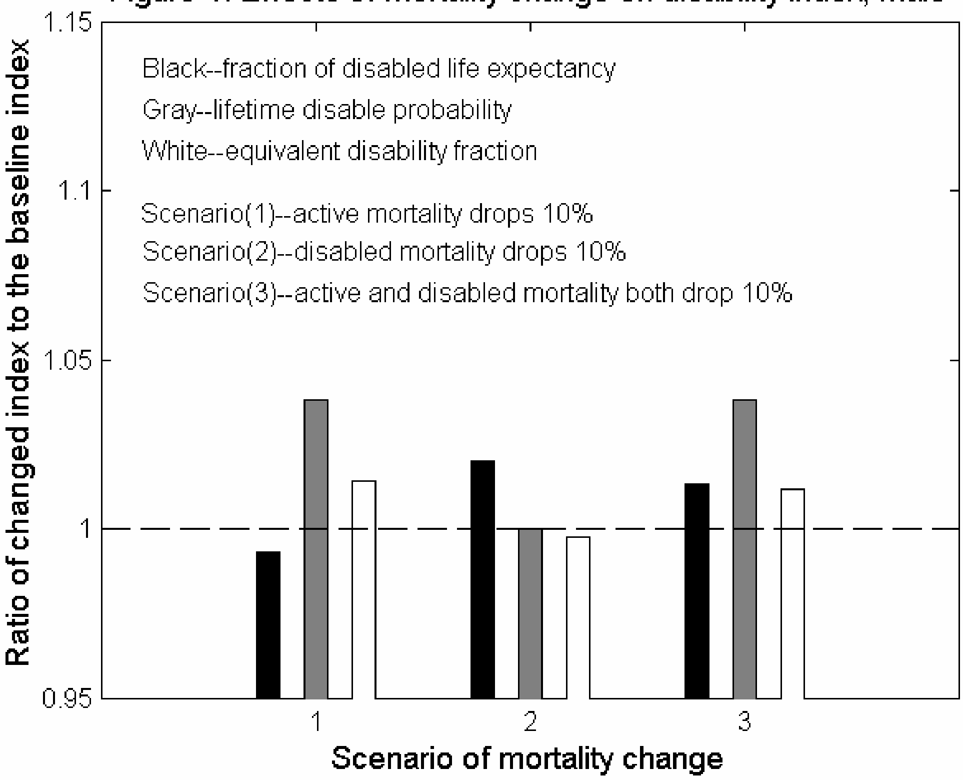


Figure 4. Effects of mortality change on disability index, male



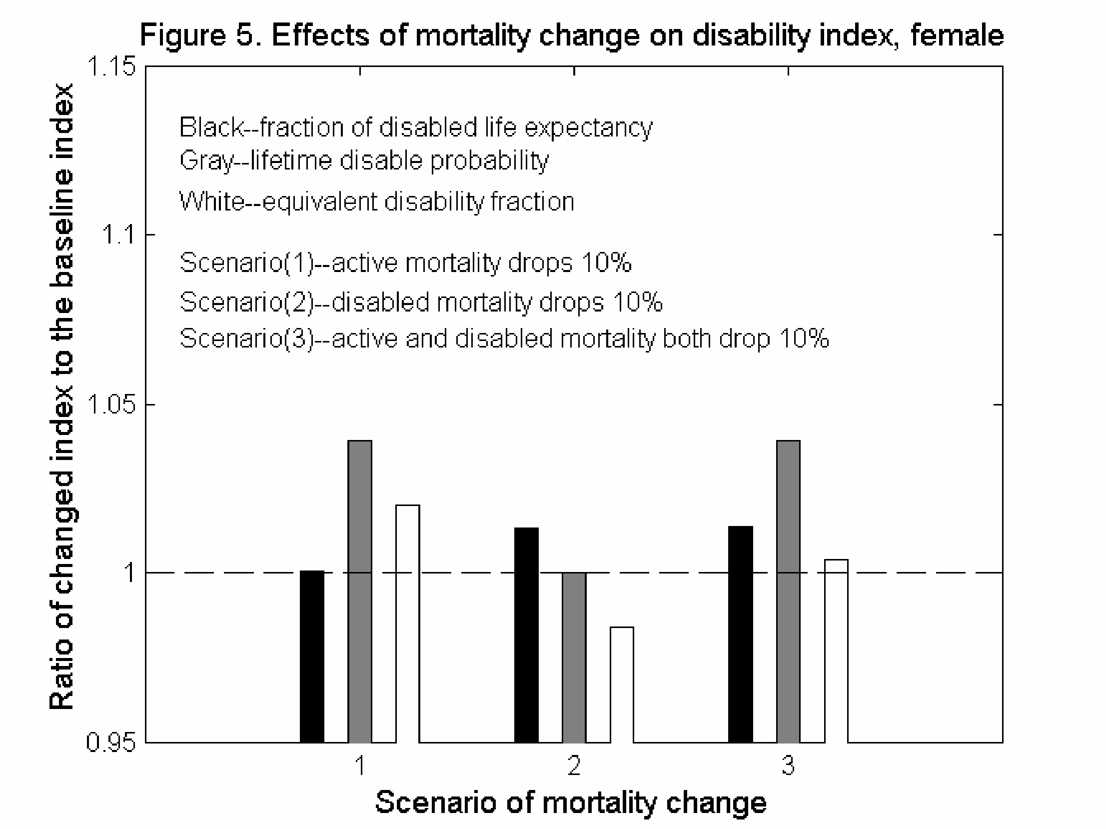


Figure 5. Effects of mortality change on disability index, female
Black--fraction of disabled life expectancy
Gray--lifetime disable probability
White--equivalent disability fraction
Scenario(1)--active mortality drops 10%
Scenario(2)--disabled mortality drops 10%
Scenario(3)--active and disabled mortality both drop 10%
Ratio of changed index to the baseline index
1.15
1.1
1.05
1
0.95
1
2
3
Scenario of mortality change